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Properties of a strongly focused Gaussian beam with an off-axis vortex

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ABSTRACT

The intensity distribution and the phase properties, especially the Gouy phase and the phase singularities are studied in a strongly focused Gaussian beam with an off-axis vortex. The symmetry relation of the focused field is also derived. It is found that the off-axis vortex induces a rotation of the field pattern, the transverse focal shift, and the asymmetric distribution of the phase singularities. Our results also show that the initial position of the off-axis vortex in the incident beam strongly influences the distance of the transverse focal shift, but does not have an effect on the Gouy phase along the central axis.

1. Introduction

Optical vortices, also called phase singularities in an optical wave field have been studied a lot [1–4] for their many potential applications, like in optical trapping [5], microscopy [6,7] and wireless communication [8]. Because of the special properties, such as the rotation of vortices with the same charge in the far field [9], off-axis vortices nested in a beam have drawn much attention in recent years. Investigations on the trajectories of vortices have been done in graded-index media [10] and for a canonically launched vortex dipole [11]. The vortices behavior, like attraction and annihilation have been examined for two vortices with the same topological charge and with the opposite charge in a tightly focused field [12]. The propagation of a Gaussian beam with vortices with arbitrary topological charges through a tilted lens was studied in [13]. In [14] optical vortices were generated by three different types of custom-designed wavefronts in the experiment and the propagation was discussed. The dynamics of vortices was also examined in a nonuniform Bose-Einstein condensate [15]. However, except [12] most works concentrate on the propagation properties in the scalar case. When a field is strongly focused, the scalar description is not sufficient and a vector analysis must be adopted. For a better understanding of the nature of the off-axis vortices in propagation, we choose to study the properties of off-axis vortices in a high numerical aperture (NA) system.

The Gouy phase, as a fundamental property in a focused field, since its first observation in 1890s [16,17], has been examined in a wide range of studies [18–22]. Although this phase anomaly plays a crucial role in many applications, such as in mode conversion [23], optical tweezers [24], propagation dynamics of optical vortices [25] and interference microscopy [26], the Gouy phase is not always taken into consideration when a strongly focused field is studied.

In the present article, the Gouy phase as well as the intensity and phase behaviors of a Gaussian beam with an off-axis vortex in a high NA system is analyzed. The Richards-Wolf vectorial model is used to derive the expressions of the focused field. As we will demonstrate, the parameters which characterize the off-axis vortex strongly influence the phase behavior and the intensity distribution of the focused field, especially the transverse focal shift, while the Gouy phase and phase singularities exhibit different behaviors for the three components of the field.

2. Theory

2.1. Focused field with multi-vortices at arbitrary positions

Assume there are N vortices of charge m_k located at $r = r_k$, $\phi = \phi_k$, embedded in a Gaussian beam. According to [9] the amplitude distribution of the electric field at the beam waist ω_0 can be expressed as

$$V_0(r, \phi) = \prod_{k=1}^{N} e^{-r^2/\omega_0^2}(re^{\pm i\phi} - r_k e^{\pm i\phi_k})^{|m_k|}, \tag{1}$$

with r the radial distance and ϕ the azimuthal angle. When m_k is positive, the sign of ϕ and ϕ_k is positive and vice versa.

Let us consider a Gaussian beam with the electric field polarized along the x-direction and the electric field amplitude described by Eq. (1). When such a beam is incident upon an aplanatic, high numerical aperture focusing system (see Fig. 1) of focal length f with a semi-aperture angle α (here we assume that the entrance plane of the

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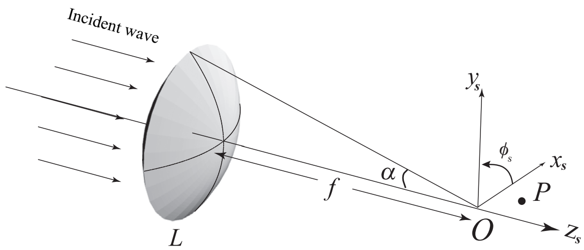

Fig. 1. Illustration of a high-numerical-aperture system. The origin O of a Cartesian coordinate system is taken at the geometrical focus.

focusing system is coincident with the waist plane of the Gaussian beam), the electric field in the focal region at the observation point P can be expressed using the Richards-Wolf vectorial diffraction model [27] as

$$E(\rho_s, z_s, \phi_s) = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = -\frac{\mathrm{i}k}{2\pi}\int_0^{\alpha}\int_0^{2\pi} f\ V_0(\theta, \phi)\sqrt{\cos\theta}\sin\theta \times \begin{bmatrix} \cos\theta + \sin^2\phi(1-\cos\theta) \\ (\cos\theta - 1)\cos\phi\sin\phi \\ -\sin\theta\cos\phi \end{bmatrix} e^{\mathrm{i}kz_s\cos\theta} e^{\mathrm{i}k\rho_s\sin\theta\cos(\phi-\phi_s)}\mathrm{d}\phi\mathrm{d}\theta, \tag{2}$$

with the wave number k is $2\pi/\lambda$ and λ is the wavelength. In this equation (ρ_s, ϕ_s, z_s) are the cylindrical coordinates in image space, where at the origin O (in Fig. 1) $\rho_s = z_s = 0$. $V_0(\theta, \phi) = V_0(r, \phi)$ since $r = f\sin\theta$ according to the sine condition in this focusing system. It is also convenient to use the dimensionless Lommel variables u, v [27] to describe the position of an observation point instead of z_s and ρ_s, namely:

$$u = kz_s\sin^2\alpha, \tag{3}$$

$$v = k\rho_s\sin\alpha. \tag{4}$$

Eq. (2) can be rewritten as:

$$E(u, v, \phi_s) = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = -\frac{\mathrm{i}k}{2\pi}\int_0^{\alpha}\int_0^{2\pi} f\ V_0(\theta, \phi)\sqrt{\cos\theta}\sin\theta \times \begin{bmatrix} \cos\theta + \sin^2\phi(1-\cos\theta) \\ (\cos\theta - 1)\cos\phi\sin\phi \\ -\sin\theta\cos\phi \end{bmatrix} e^{\mathrm{i}\frac{u\cos\theta}{\sin^2\alpha}} e^{\mathrm{i}\frac{v\sin\theta}{\sin\alpha}\cos(\phi-\phi_s)}\mathrm{d}\phi\mathrm{d}\theta. \tag{5}$$

2.2. Gouy phase

The Gouy phase, δ describes how the phase of an actual focused field differs from that of a non-diffracted wave (see [28], Sec.8.8.4). In a strongly focused field, as expressed by Eq. (5), there are three electric components and for each individual component a Gouy phase can be defined as

$$\delta_x(u, v, \phi_s) = \psi_x(u, v, \phi_s) - \mathrm{sign}[u]kR, \tag{6}$$

$$\delta_y(u, v, \phi_s) = \psi_y(u, v, \phi_s) - \mathrm{sign}[u]kR, \tag{7}$$

$$\delta_z(u, v, \phi_s) = \psi_z(u, v, \phi_s) - \mathrm{sign}[u]kR, \tag{8}$$

here $\psi_j (j = x, y, z)$ denotes the phase of e_j and R is the distance from the observation point P to the focus O, i.e.

$$kR = k\sqrt{z_s^2 + \rho_s^2} = \frac{1}{\sin\alpha}\sqrt{\frac{u^2}{\sin^2\alpha} + v^2}, \tag{9}$$

and sign denotes the sign function, namely

$$\mathrm{sign}[u] = \begin{cases} -1 & \text{when} u < 0, \\ 1 & \text{when} u \geq 0. \end{cases} \tag{10}$$

3. An off-axis vortex embedded in the Gaussian beam

Consider an off-axis vortex with $m = -1$ nested in the Gaussian beam at position $r_1 = a$, $\phi_1 = 0$. Eq. (1) changes into $V_0(r, \phi) = e^{-r^2/\omega_0^2}(re^{-\mathrm{i}\phi} - a)$. Substituting this expression into Eq. (5) and applying the following identity:

$$\int_0^{2\pi} e^{\mathrm{i}n\phi} e^{\mathrm{i}\frac{v\sin\theta}{\sin\alpha}\cos(\phi-\phi_s)}\mathrm{d}\phi = 2\pi \mathrm{i}^n e^{\mathrm{i}n\phi_s} J_n\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{11}$$

with J_n being the first kind Bessel function of order n, we can calculate the electric field near the focus as:

$$e_x(u, v, \phi_s) = -\mathrm{i}k\int_0^{\alpha} P(\theta)\ (I_{x0} + I_{x1} + I_{x2} + I_{x3})\ e^{\mathrm{i}\frac{u\cos\theta}{\sin^2\alpha}}\mathrm{d}\theta, \tag{12}$$

$$e_y(u, v, \phi_s) = -\mathrm{i}k\int_0^{\alpha} P(\theta)\ (I_{y1} + I_{y2} + I_{y3})\ e^{\mathrm{i}\frac{u\cos\theta}{\sin^2\alpha}}\mathrm{d}\theta, \tag{13}$$

$$e_z(u, v, \phi_s) = -\mathrm{i}k\int_0^{\alpha} P(\theta)\ (I_{z0} + I_{z1} + I_{z2})\ e^{\mathrm{i}\frac{u\cos\theta}{\sin^2\alpha}}\mathrm{d}\theta, \tag{14}$$

where

$$P(\theta) = f\ e^{-f^2\sin^2\theta/\omega_0^2}\sqrt{\cos\theta}\sin\theta, \tag{15}$$

and

$$I_{x0}(\theta; v, \phi_s) = -\frac{1}{2}a(1+\cos\theta)\ J_0\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{16}$$

$$I_{x1}(\theta; v, \phi_s) = \mathrm{i}f\sin\theta\left[\frac{1}{2}(1+\cos\theta)e^{-\mathrm{i}\phi_s} + \frac{1}{4}(\cos\theta - 1)e^{\mathrm{i}\phi_s}\right] J_1\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{17}$$

$$I_{x2}(\theta; v, \phi_s) = \frac{1}{2}a(\cos\theta - 1)\cos 2\phi_s\ J_2\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{18}$$

$$I_{x3}(\theta; v, \phi_s) = -\mathrm{i}\frac{1}{4}f\sin\theta(\cos\theta - 1)e^{-\mathrm{i}3\phi_s}\ J_3\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{19}$$

$$I_{y1}(\theta; v, \phi_s) = \frac{1}{4}f\sin\theta(\cos\theta - 1)e^{\mathrm{i}\phi_s}\ J_1\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{20}$$

$$I_{y2}(\theta; v, \phi_s) = \frac{1}{2}a(\cos\theta - 1)\sin 2\phi_s\ J_2\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{21}$$

$$I_{y3}(\theta; v, \phi_s) = \frac{1}{4}f\sin\theta(\cos\theta - 1)e^{-\mathrm{i}3\phi_s}\ J_3\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{22}$$

$$I_{z0}(\theta; v, \phi_s) = -\frac{1}{2}f\sin^2\theta\ J_0\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{23}$$

$$I_{z1}(\theta; v, \phi_s) = \mathrm{i}a\sin\theta\cos\phi_s\ J_1\left(\frac{v\sin\theta}{\sin\alpha}\right), \tag{24}$$

$$I_{z2}(\theta; v, \phi_s) = \frac{1}{2}f\sin^2\theta e^{-\mathrm{i}2\phi_s}\ J_2\left(\frac{v\sin\theta}{\sin\alpha}\right). \tag{25}$$

It follows from Eqs. (12) to (25) that these three components obey the following symmetry relations:

$$e_x^*(u, v, \phi_s) = -e_x(-u, v, \pi - \phi_s), \tag{26}$$

$$e_y^*(u, v, \phi_s) = e_y(-u, v, \pi - \phi_s), \tag{27}$$

$$e_z^*(u, v, \phi_s) = -e_z(-u, v, \pi - \phi_s), \tag{28}$$