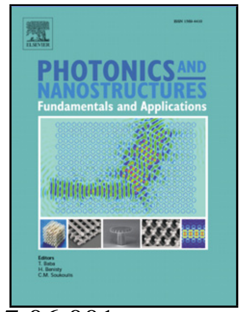

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Linear electromagnetic wave equations in bulk materials

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Abstract

After a short review of microscopic electrodynamics in materials, we investigate the relation of the microscopic dielectric tensor to the current response tensor and to the full electromagnetic Green function. Subsequently, we give a systematic overview of microscopic electromagnetic wave equations in bulk materials, which can be formulated in terms of the microscopic dielectric tensor.

Keywords: electrodynamics, wave equations in materials, dielectric tensor

1. Introduction

The derivation of linear electromagnetic wave equations *in materials* is both conceptually and practically of a fundamental importance. On the practical side, it is evident that wave equations in materials form the basis for the description of pulse propagation in media, a topic which is even part of the standard textbook literature (see e.g. [1, Ch. 7] or [2, Ch. 18 and Ch. 19]). On the theoretical side, wave equations in media allow one to relate the *speed of light in materials* and hence the *refractive index* to electromagnetic response properties such as the *dielectric function* [3–17]. Although this is in principle incontrovertible, it is less evident that these problems require a re-investigation from the viewpoint of modern *ab initio materials physics*. This concerns, in particular, the standard wave equation in media and the standard relation for the refractive index (see the paradigmatic discussion in Ref. [18]). In this article, we resume this line of research by a systematic discussion of linear electromagnetic wave equations in bulk materials. For this purpose, we rely on a microscopic approach to electrodynamics of media, which is well-established in first-principles materials science and in plasma physics (see [19–26] for modern textbooks), and which is axiomatized by the Functional Approach to electrodynamics of media (see Refs. [18, 27–32]). In particular, the present analysis clarifies on the most fundamental level the rôle of the *ab initio* dieletric tensor in linear microscopic electromagnetic wave equations, and will thus ultimately contribute to a first-principles description of the propagation of photons in media or, more generally, all sorts of nanostructures.

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Concretely, this article is organized as follows: After assembling in Sct. 2 a number of essential facts and useful formulae from classical electrodynamics, we proceed in Sct. 3 with a review of microscopic electrodynamics of materials. This then allows us to derive electromagnetic wave equations in bulk materials systematically in Sct. 4. Finally, Sct. 5 investigates these findings in the practically important case of isotropic materials.

2. Basic notions of classical electrodynamics

2.1. Cartesian projector formalism

We begin by introducing some technical definitions, which will be frequently used throughout this article. The *Cartesian longitudinal* and *transverse projectors* are defined as operators acting on the three-dimensional space [27, § 2.1], i.e.,

$$(P_{\mathrm{L}})_{ij}(\boldsymbol{k}) = \frac{k_i k_j}{|\boldsymbol{k}|^2}\,, \tag{2.1}$$

$$(P_{\mathrm{T}})_{ij}(\boldsymbol{k}) = \delta_{ij} - \frac{k_i k_j}{|\boldsymbol{k}|^2}\,. \tag{2.2}$$

With these, any (3×3) *Cartesian tensor* $C_{ij}(\boldsymbol{k})$ can be uniquely decomposed into four contributions,

$$\overleftrightarrow{C}(\boldsymbol{k}) = \overleftrightarrow{C}_{\mathrm{LL}}(\boldsymbol{k}) + \overleftrightarrow{C}_{\mathrm{LT}}(\boldsymbol{k}) + \overleftrightarrow{C}_{\mathrm{TL}}(\boldsymbol{k}) + \overleftrightarrow{C}_{\mathrm{TT}}(\boldsymbol{k})\,, \tag{2.3}$$

which are respectively given by

$$\overleftrightarrow{C}_{\mathrm{LL}}(\boldsymbol{k}) = \overleftrightarrow{P}_{\mathrm{L}}(\boldsymbol{k})\,\overleftrightarrow{C}(\boldsymbol{k})\,\overleftrightarrow{P}_{\mathrm{L}}(\boldsymbol{k})\,, \tag{2.4}$$

$$\overleftrightarrow{C}_{\mathrm{LT}}(\boldsymbol{k}) = \overleftrightarrow{P}_{\mathrm{L}}(\boldsymbol{k})\,\overleftrightarrow{C}(\boldsymbol{k})\,\overleftrightarrow{P}_{\mathrm{T}}(\boldsymbol{k})\,, \tag{2.5}$$

$$\overleftrightarrow{C}_{\mathrm{TL}}(\boldsymbol{k}) = \overleftrightarrow{P}_{\mathrm{T}}(\boldsymbol{k})\,\overleftrightarrow{C}(\boldsymbol{k})\,\overleftrightarrow{P}_{\mathrm{L}}(\boldsymbol{k})\,, \tag{2.6}$$

$$\overleftrightarrow{C}_{\mathrm{TT}}(\boldsymbol{k}) = \overleftrightarrow{P}_{\mathrm{T}}(\boldsymbol{k})\,\overleftrightarrow{C}(\boldsymbol{k})\,\overleftrightarrow{P}_{\mathrm{T}}(\boldsymbol{k})\,. \tag{2.7}$$