Photonics and Nanostructures – Fundamentals and Applications 24 (2017) 63–68

Contents lists available at ScienceDirect

Photonics and Nanostructures – Fundamentals and Applications

journal homepage: www.elsevier.com/locate/photonics

Photonic band structures of one dimensional multilayered dielectric-magnetic photonic crystals

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ARTICLE INFO

Article history:
Received 10 September 2016
Received in revised form 25 January 2017
Accepted 9 March 2017
Available online 19 March 2017

Keywords:
Multilayered thin films
One-dimensional photonic crystal
Dielectric-magnetic photonic crystal
Transfer matrix method
Band diagram

ABSTRACT

The one dimensional dielectric-magnetic photonic crystals (PCs) are materials with periodically varying permittivity and periodically varying permeability along the same direction. These PCs offer more degrees of freedom in designing photonic crystals. Most dielectric-magnetic PCs are fabricated with only two layers in each repeating unit cell. However, more than two layers in each periods are phenomenologically richer. This article presents the generalized formulation for the computation of Bloch wavenumbers and photonic bands of multilayered dielectric-magnetic PC based on the transfer matrix method. The formulation is valid for the lossy and lossless constituent materials. The numerical results for a dielectric only, magnetic only, and a dielectric-magnetic PC are also presented and discussed for the axial as well as oblique propagation.

1. Introduction

Ever since the seminal paper by Yablonovich [1] on the photonic crystals (PCs), the research and applications of PCs have exploded [2]. However, almost all of the research and applications have focused on dielectric PCs which have a periodically varying permittivity in one, two, or three dimensions, and a constant permeability. This is due to the fact that dielectric materials are available with a wide range of relative permittivity over a spectral regime from the radio waves to the ultraviolet. The dielectric PCs [3] basically provide electromagnetic wave guiding over selective spectral regime [4] that can be realized over a wide range of electromagnetic spectrum including optical, microwave, and millimeter-wave frequencies [5], and find applications in PC waveguides, cavities [2], optical fibers [6], and zero-index metamaterials [7,8], to name a few.

In the recent past, attention is being paid to exploit another degree of freedom in designing the PCs, that is the periodic variation of the permeability. Such photonic crystals are called magnetic PCs or magnetophotonic crystals [9]. The late attention to the magnetic PCs is due to the fact that the appreciable magnetic effects are observed mostly at low frequencies [13]. However, with the advent of nano-engineered magnetic materials at high-frequencies [11], the interest in magnetic PCs is increasing. The fabrication of a magnetic material at optical frequencies involve metallic nanostructures [11]. Let us note that the term magnetophotonic crystal is also used for those dielectric PCs that have relative permeability of unity, but a relative permittivity tensor that is dependent on the externally applied low-frequency magnetic field [12,10].

One dimensional PCs with alternating layers of a metal and a dielectric material have also gained a lot of attention due to their strong non-local response in the effective-medium regime, where the thicknesses of individual layers are much smaller than the operating wavelength [14,15]. These metal-dielectric PCs can also lead to effective epsilon-near-zero material and a strong effective chirality [16]. Furthermore, these PCs find applications in subwavelength imagining [17] and diffractionless propagation [18].

With the research in multilayered PCs increasing, there is a need for developing theoretical models for computation of its optical properties. There are several reports in the literature where the band diagrams of one dimensional dielectric-magnetic PC have been computed [11,14,15,19,20]. However, they all used a PC with alternating layers of two different materials in each unit cell in ABABAB fashion. In this paper, we present the general formulation for arbitrary number of layers in each unit cell with different permittivity and permeability in each layer. Multilayered unit cell offer much more control over the characteristics of a one dimensional PC. The theoretical formulation for both the TE and TM polarization states are provided in Section 2. This formulation is valid for a PC with metallic as well as lossy dielectric layers. The representative numerical results for a PC with six layers in each unit cell are

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http://dx.doi.org/10.1016/j.photonics.2017.03.004

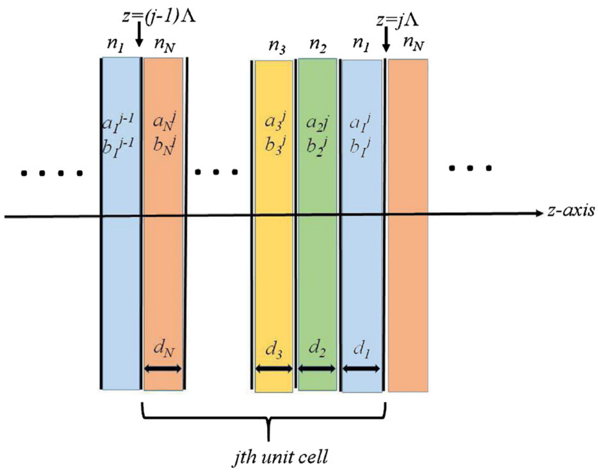

Fig. 1. Schematic of a one-dimensional photonic crystal (PC) with N layers in each unit cell. Each layer, in general, can have different relative permittivity and permeability. The wave propagation is assumed to be in the yz plane (the plane of the paper).

presented and discussed in Section 3. The concluding remarks are presented in Section 4.

2. Theoretical formulation

Consider a one-dimensional PC with N layers in each unit cell, as schematically shown in Fig. 1. Let each layer be an isotropic homogeneous dielectric-magnetic material. Without loss of generality, let us assume that the PC is periodic along the z axis and the propagation of electromagnetic waves takes place in the yz plane. Furthermore, let us assume that both the permittivity and permeability have the same period Λ. Let $\varepsilon_{r\ell}$ and $\mu_{r\ell}$ be the relative permittivity and permeability, respectively, of ℓth layer in each unit cell. Because all materials are isotropic, the transverse electric (TE) and transverse magnetic (TM) (with respect to the yz plane) polarization states decouple from each other. We present formulation below for both polarization states following the transfer matrix method given in Ref. [21] for a dielectric only, one-dimensional PC, with only two layers in each unit cell.

2.1. TE polarization state

Let us first consider the case when electric field is transverse to plane of propagation, i.e.; yz plane. For this case, only E_x, H_y, H_z are non-zero and are coupled together, as is shown by Maxwell equations.

Assuming an $e^{-i\omega t}$ time dependence and the propagation in the yz plane, the field phasors in each layer in the jth unit cell can be written as a superposition of two modes propagating along $\pm z$ axis with unknown coefficients:

$$E_x(z) = \begin{cases} a_1^{(j-1)} c^{-ik_{z1}[z-(j-1)\Lambda]} + b_1^{(j-1)} e^{ik_{z1}[z-(j-1)\Lambda]}, & (j-1)\Lambda - d_1 < z < (j-1)\Lambda \\ a_\ell^{(j)} e^{-ik_{z\ell}[z-j\Lambda+L_{\ell-1}]} + b_\ell^{(j)} e^{ik_{z\ell}[z-j\Lambda+L_{\ell-1}]}, & j\Lambda - L_\ell < z < j\Lambda - L_{\ell-1} \\ a_N^{(j)} e^{-ik_{zN}[z-(j-1)\Lambda+d_N]} + b_N^{(j)} e^{ik_{zN}[z-(j-1)\Lambda+d_N]}, & (j-1)\Lambda < z < (j-1)\Lambda + d_N, \end{cases}$$

for ℓ=1, 2, 3, . . ., $N-1$, and the wavenumber along the z axis is given by

$$k_{z\ell} = \sqrt{\left(\frac{n_\ell w}{c}\right)^2 - k_y{}^2}, \quad \ell = 1, 2, 3, \ldots, N-1,$$

where $n_\ell = \sqrt{\mu_{r\ell}\varepsilon_{r\ell}}$,

$$L_\ell = \sum_{k=1}^{\ell} d_k, \tag{1}$$

$$\Lambda = L_N = \sum_{k=1}^{N} d_k. \tag{2}$$

The standard boundary conditions at each interface between two adjacent layers demand the continuity of E_x and H_y. Faraday's law gives

$$H_y = \frac{1}{i\omega\mu_{r\ell}\mu_0}\frac{\partial E_x}{\partial z}, \tag{3}$$

in the ℓth layer. Therefore, the continuity of H_y demands

$$\frac{1}{\mu_{r\ell}}\frac{\partial E_x}{\partial z} = \frac{1}{\mu_{r(\ell+1)}}\frac{\partial E_x}{\partial z}, \tag{4}$$

where left hand side is evaluated in the ℓth layer and the right hand side in the (ℓ+1)th layer, but at the interface between the two layers.

The application of the boundary conditions (continuity of E_x and H_y) at $z=(j-1)\Lambda$ yields

$$a_N^{(j)} e^{-ik_{zN}d_N} + b_N^{(j)} e^{ik_{zN}d_N} = a_1^{(j-1)} + b_1^{(j-1)}, \tag{5}$$

$$\frac{k_{zN}}{\mu_{rN}}\left[a_N^{(j)} e^{-ik_{zN}d_N} - b_N^{(j)} e^{ik_{zN}d_N}\right] = \frac{k_{z1}}{\mu_{r1}}\left[a_1^{(j-1)} - b_1^{(j-1)}\right], \tag{6}$$

and at $z=j\Lambda - L_{\ell-1}$,

$$a_{\ell+1}^{(j)} e^{-ik_{z(\ell+1)}d_{\ell+1}} + b_{\ell+1}^{(j)} e^{ik_{z(\ell+1)}d_{\ell+1}} = a_\ell^{(j)} + b_\ell^{(j)}, \quad \ell = 1, 2, 3, \ldots, N-1, \tag{7}$$

$$\frac{k_{z(\ell+1)}}{\mu_{r_{\ell+1}}}\left[a_{\ell+1}^{(j)} e^{-ik_{z(\ell+1)}d_{\ell+1}} - b_{\ell+1}^{(j)} e^{ik_{z(\ell+1)}d_{\ell+1}}\right] = \frac{k_{z\ell}}{\mu_{r\ell}}[a_\ell^{(j)} - b_\ell^{(j)}], \quad \ell = 1, 2, 3, \ldots, N-1. \tag{8}$$

Solving Eqs. (5)–(8) simultaneously, we get

$$\begin{bmatrix} a_1^{(j-1)} \\ b_1^{(j-1)} \end{bmatrix} = \left[K_1^{TE}\right]^{-1}\left(\prod_{\ell=2}^{N} A_\ell^{TE}[K_\ell^{TE}]^{-1}\right) A_1^{TE} \begin{bmatrix} a_1^{(j)} \\ b_1^{(j)} \end{bmatrix}, \tag{9}$$

where

$$K_\ell^{TE} = \begin{bmatrix} 1 & 1 \\ \dfrac{ik_{z\ell}}{\mu_{r\ell}} & \dfrac{-ik_{z\ell}}{\mu_{r\ell}} \end{bmatrix},$$

and

$$A_\ell^{TE} = \begin{bmatrix} e^{ik_{z\ell}d_\ell} & e^{-ik_{z\ell}d_\ell} \\ \dfrac{ik_{z\ell}}{\mu_{r\ell}} e^{ik_{z\ell}d_\ell} & \dfrac{-ik_{z\ell}}{\mu_{r\ell}} e^{-ik_{z\ell}d_\ell} \end{bmatrix}.$$

A transfer matrix connecting the fields in the corresponding layers in the jth and $(j-1)$th unit cells can be defined as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [K_1^{TE}]^{-1}\left(\prod_{\ell=2}^{N} A_\ell^{TE}[K_\ell^{TE}]^{-1}\right) A_1^{TE}, \tag{10}$$

so that Eq. (9) becomes

$$\begin{bmatrix} a_1^{(j-1)} \\ b_1^{(j-1)} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} a_1^{(j)} \\ b_1^{(j)} \end{bmatrix}. \tag{11}$$