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Conditional electron confinement in graphene via smooth magnetic fields

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ABSTRACT

In this article we discuss confinement of electrons in graphene via smooth magnetic fields which are finite everywhere on the plane. We shall consider two types of magnetic fields leading to systems which are conditionally exactly solvable and quasi exactly solvable. The bound state energies and wavefunctions in both cases have been found exactly.

1. Introduction

In recent years graphene which is a sheet of carbon atom in honeycomb lattice [1–3] has drawn widespread attention because of its possible applications in various devices. The dynamics of charge carriers or electrons in graphene is described by the (2 + 1) dimensional massless Dirac equation, except that the electrons move with the much smaller Fermi velocity $v_F = 10^6$ m/s instead of the velocity of light c. For graphene to have practical applications one of the most important problem is controlling or confining the electrons. Attempts have been made to confine electrons e.g, by using position dependent mass [4], modulating Fermi velocity [5,6], electrostatic fields or magnetic fields. However, confinement using electrostatic fields is usually difficult although zero energy states [7–11] and sometimes some states of non zero energy [12] can be found using different field configurations. On the other hand magnetic confinement of electrons has been studied by many authors. For example, square well magnetic barrier [13,14], radial magnetic field [15], decaying gaussian magnetic field [16], hyperbolic magnetic fields [17], inhomogeneous magnetic fields [18–22], one dimensional magnetic fields leading to solvable systems [23], etc. have been used to create bound states in graphene. In particular, of the different types of magnetic fields mentioned above, there are some smooth inhomogeneous magnetic fields [19–21] for which the pseudospinor components satisfy equations with quasi exactly solvable effective potentials [24]. In this context, it may be noted that inhomogeneous magnetic field profiles can be produced in many ways e.g, using ferromagnetic materials [25], non planar substrate [26], integrating superconducting elements [27] etc. In the present paper, our objective is to search for smooth everywhere finite magnetic fields which produce conditionally exactly solvable effective potentials [28,29] i.e, potentials which admit exact solutions when parameter(s) of the model assume particular values. More precisely, it will be shown that the electrons remain confined for certain values of the magnetic quantum number while for other values of hte magnetic quantum number they enter the deconfining phase. We shall also explore the possibility of obtaining quasi exactly solvable systems when some of the constraints on the parameters are relaxed. The organization of the paper is as follows: in Section 2 we shall present the formalism; in Section 3 we shall obtain several magnetic fields which leads to conditionally exactly solvable systems; in Section 4 we shall examine under what conditions the magnetic fields produce quasi exactly solvable systems and finally Section 5 is devoted to a conclusion.

2. Formalism

The dynamics of quasi particles in graphene is governed by the Hamiltonian

$$\widehat{H} = v_F \vec{\sigma}\cdot\vec{\pi} = v_F \vec{\sigma}\cdot(\vec{p} + \vec{A}) = v_F \begin{pmatrix} 0 & \widehat{\Pi}_- \\ \widehat{\Pi}_+ & 0 \end{pmatrix}, \tag{1}$$

where v_F is the Fermi velocity, $\sigma = (\sigma_x, \sigma_y)$ are Pauli matrices, and

$$\widehat{\Pi}_\pm = \hat{\pi}_x \pm i\hat{\pi}_y = (\hat{p}_x + A_x) \pm i(\hat{p}_y + A_y). \tag{2}$$

We now choose the vector potentials to be of the form

$$A_x = yf(r), \quad A_y = -xf(r) \tag{3}$$

where the specific form of the function $f(r)$ will be chosen later. With the above choice of the vector potentials, the magnetic field is given by

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$$B_z = -2f(r) - rf'(r). \tag{4}$$

The eigenvalue equation

$$\widehat{H}\psi = E\psi,$$

where $\psi = (\psi_1, \psi_2)^T$ is a two component pseudospinor, can be written as

$$\widehat{\Pi}_-\psi_2 = \epsilon\psi_1, \tag{5}$$

$$\widehat{\Pi}_+\psi_1 = \epsilon\psi_2, \tag{6}$$

where $\epsilon = E/v_F$. Now eliminating ψ_1 in favor of ψ_2 (and vice-versa), the equations for the components can be written as

$$\widehat{\Pi}_-\widehat{\Pi}_+\psi_1 = \epsilon^2\psi_1, \tag{7}$$

$$\widehat{\Pi}_+\widehat{\Pi}_-\psi_2 = \epsilon^2\psi_2. \tag{8}$$

Since the magnetic field is a radial one, the pseudospinor components can be taken as

$$\psi_1 = e^{im\theta}r^{-1/2}\phi_1(r), \quad \psi_2 = e^{i(m+1)\theta}r^{-1/2}\phi_2(r), \quad m = 0, \pm 1, \pm 2, \ldots, \tag{9}$$

where m is the magnetic quantum number. Then eigenvalue equations for the components can be written as

$$\left[-\frac{d^2}{dr^2} + \frac{m^2 - \frac{1}{4}}{r^2} + r^2f^2 - 2(m+1)f - rf'\right]\phi_1 = \epsilon^2\phi_1, \tag{10}$$

$$\left[-\frac{d^2}{dr^2} + \frac{(m+1)^2 - \frac{1}{4}}{r^2} + r^2f^2 - 2mf + rf'\right]\phi_2 = \epsilon^2\phi_2. \tag{11}$$

Before closing this section, we note that the intertwining relations (5) and (6) can also be written in terms of polar coordinates and are given by

$$\left(\frac{\partial}{\partial r} - \frac{m + \frac{1}{2}}{r} + rf\right)\phi_1 = i\epsilon\phi_2,$$
$$\left(\frac{\partial}{\partial r} + \frac{m + \frac{1}{2}}{r} - rf\right)\phi_2 = i\epsilon\phi_1. \tag{12}$$

The set of intertwining relations (12) is particularly important since knowing solution of one of the two Eqs. (10) or (11), the other can be obtained through the above relations.

3. Conditionally exactly solvable magnetic fields

Here we shall consider several conditionally exactly solvable magnetic field profiles i.e, magnetic fields for which all or some bound state solutions can be found *only* when the parameters of the model assume some specific values. To this end we choose the function $f(r)$ to be of the form

$$f(r) = \frac{\lambda}{2} + \sum_{i=1}^{N}\frac{2g_i}{1 + g_ir^2}, \quad \lambda > 0,\ g_1, g_2, \ldots, g_N > 0. \tag{13}$$

Then the resulting magnetic field is given by

$$B_z(r) = -\lambda - \sum_{i=1}^{N}\frac{4g_i}{(1 + g_ir^2)^2}. \tag{14}$$

From Eq. (14) it can be observed that the magnetic field is everywhere finite with a maximum value of $-\lambda$ and a minimum of $-\lambda - 4\sum_{i=0}^{N} g_i$. We shall now consider different values of N and examine if the corresponding magnetic field can support bound states when the parameters assume some particular values.

3.1. N = 1

In this case the magnetic field becomes

$$B_z(r) = -\lambda - \frac{4g_1}{(1 + g_1r^2)^2}, \tag{15}$$

and the profile of this field can be seen in Fig. 1.

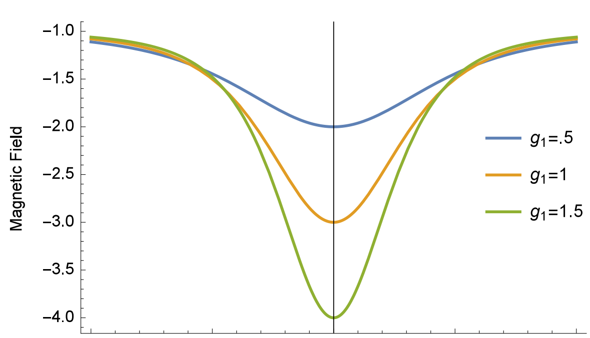

Fig. 1. Magnetic field profile for $N = 1$, $\lambda = 1$.

Then, from (10) and (11) the equations for the components ϕ_1 and ϕ_2 can be obtained as

$$\left[-\frac{d^2}{dr^2} + \frac{m^2 - \frac{1}{4}}{r^2} + \frac{\lambda^2r^2}{4} - \frac{2Z - 4g_1}{1 + g_1r^2} - \frac{8g_1}{(1 + g_1r^2)^2}\right]\phi_1 = [\epsilon^2 + \lambda(m-1)]\phi_1, \tag{16}$$

$$\left[-\frac{d^2}{dr^2} + \frac{(m+1)^2 - \frac{1}{4}}{r^2} + \frac{\lambda^2r^2}{4} - \frac{2Z}{1 + g_1r^2}\right]\phi_2 = [\epsilon^2 + \lambda(m-2)]\phi_2, \tag{17}$$

where $Z = 2\,mg_1 + \lambda$.

Conditional exact solutions: Let us now consider Eq. (17) for the lower component. This equation can be interpreted as the radial Schrödinger equation for a particle moving in a two dimensional nonpolynomial oscillator potential. Next, we choose the parameter g_1 in such a way that the nonpolynomial part vanishes i.e,[1]

$$Z = 0 \Rightarrow g_1 = -\lambda/2\,m. \tag{18}$$

Now recalling that g_1 and λ are always positive, the admissible values of m are $m < 0$. With g_1 as given above, Eq. (17) becomes the radial Schrödinger equation for the two-dimension isotropic harmonic oscillator:

$$\left[-\frac{d^2}{dr^2} + \frac{(M-1)^2 - \frac{1}{4}}{r^2} + \frac{\lambda^2r^2}{4}\right]\phi_2 = [\epsilon^2 - \lambda(M+2)]\phi_2,\ M = -m. \tag{19}$$

It may be pointed out the effective potential becomes that of the radial harmonic oscillator only when g_1 assumes the particular value given by (18). The eigenvalues and the corresponding wave functions of (19) are standard and are given by:

$$E_{n,M} = \pm v_F\sqrt{2\lambda(n + M + 1)},\ \ n = 0, 1, 2, \ldots, M = 1, 2, \cdots \tag{20}$$

$$\phi_2(r) \sim r^{M-1}e^{-\lambda r^2/4}\mathcal{L}_n^{M-1}(\lambda r^2/2), \tag{21}$$

where $\mathcal{L}_n^M(x)$ is the associated Laguerre polynomial. Then, the lower component of the pseudospinor wave function ψ_2 is

[1] Note that if $\lambda < 0$, solutions can be obtained in a similar way for the sector $m > 0$.