

Condensate oscillations in a Penrose tiling lattice

Z. Akdeniz^{a,*}, P. Vignolo^{b,*}

^a Faculty of Science and Letters, Piri Reis University, 34940 Tuzla, Istanbul, Turkey

^b Université Côte d'Azur, CNRS, Institut de Physique de Nice, 1361 route des Lucioles, 06560 Valbonne, France

ARTICLE INFO

Keywords:

Dynamic properties of condensates
Quantum transport
Quasicrystals

ABSTRACT

We study the dynamics of a Bose-Einstein condensate subject to a particular Penrose tiling lattice. In such a lattice, the potential energy at each site depends on the neighbour sites, accordingly to the model introduced by Sutherland [16]. The Bose-Einstein wavepacket, initially at rest at the lattice symmetry center, is released. We observe a very complex time-evolution that strongly depends on the symmetry center (two choices are possible), on the potential energy landscape dispersion, and on the interaction strength. The condensate-width oscillates at different frequencies and we can identify large-frequency reshaping oscillations and low-frequency rescaling oscillations. We discuss in which conditions these oscillations are spatially bounded, denoting a self-trapping dynamics.

1. Introduction

Quasicrystals have been fascinating the scientific community since their discovery [1]. At halfway between periodic crystals and disordered lattices, quasicrystals present some amazing physical properties, as their fingerprint, the Bragg spectrum, that is point-like as in periodic crystals but with symmetries that are forbidden by periodicity.

The geometrical structure of quasicrystals obviously has an impact too on their electronic spectrum [2–5], whose gaps are closely related to the Bragg spectrum [6]. The spectral properties of quasicrystals are well known in one dimension (1D) [7,8]. In 1D, the spectrum for a Schrödinger operator subjected to a quasi-periodic potential or living on a quasicrystal is a Cantor set [9,10], but the nature of the states can be very different. In Aubry-André lattices, the eigenstates can be extended, exponentially localized or critical, depending on the strength of the quasiperiodic potential [9]. On the other side, for quasicrystals obtained with recurrence rules as for the a Fibonacci tiling, no state can be exponentially localized [11]. Clearly the spectrum and the nature of the eigenstates influence the system transport properties, and if the Aubry-André lattice can behave as an insulator [12], Fibonacci tilings can show high transmission coefficients for self-similar wave-functions [13,14].

In higher dimensions, the state of play of the theoretical picture is less advanced, even if, for the case of Penrose quasicrystals, some exact results are known. We mention here the possibility, by varying the Hamiltonian parameterization, to have localized states at finite support at the center of the spectrum [15], or a critical ground state at vanishing energy [16]. Geometrical frustrated states could be observed

too [17].

In this paper we study the dynamics of a (non)interacting Bose-Einstein condensate (BEC) in the presence of a special model of a Penrose lattice where critical properties can be driven by varying the Hamiltonian parameters [16]. Hereafter we will name this potential the Sutherland-Penrose lattice (SPL). The physics of BECs confined in two dimensional quasiperiodic lattices with a five-fold symmetry was already studied in [18]. In such a system, that it can be created via the interference of five laser beams [19] it is possible to observe the transition from ballistic expansion to localized regimes, in the absence and in the presence of the interactions, as a function of the strength of the quasi-periodic potential. In the presence of the SPL, we observe a richer dynamics. The expansion of a wavepacket is accompanied by size oscillations whose frequency depend on the interaction strength. We observe high-frequency reshaping oscillations and low-frequency rescaling oscillations. Depending on the initial conditions, not-surprisingly, interactions can induce the self-trapping of the BEC.

The paper is organized as following. In Section 2 we remind the Sutherland construction to define the SPL site energies. At variance with [16], we consider both the two possible symmetry centers. In Section 3 we show that the expansion of a BEC condensate initially at rest and localized at the lattice center strongly depends, not only on the symmetry center choice, but also on the potential energy dispersion and on the interaction strength. Some concluding remarks are given in Section 4.

* Corresponding authors.

E-mail addresses: zehra.akdeniz@pirireis.edu.tr (Z. Akdeniz), Patrizia.Vignolo@inphyni.cnrs.fr (P. Vignolo).

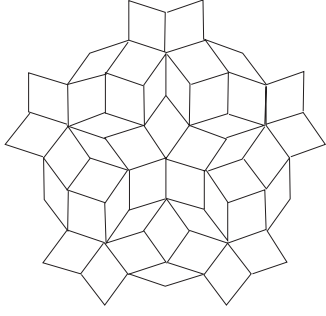


Fig. 1. Rhombus Penrose tiling.

2. The Sutherland-Penrose-lattice model

We consider the Penrose lattice introduced previously by Sutherland [16]. The starting point of such a model is a Penrose rhombus tiling (see Fig. 1): the vertices correspond to the lattice sites and the rhombuses edges correspond to the possibility for a particle to hop among the sites. All the hopping terms t are equal, while the site energies ε_i depend on the type of vertex.

One can identify 8 types of vertex that can be picked out by looking at the tiles surrounding these vertices (see for instance [20]). The corresponding site energies ε_i are chosen in order to ensure that the ground state of the single-particle tight-binding Hamiltonian

$$H = - \sum_{\langle ij \rangle} t (|i\rangle\langle j| + |j\rangle\langle i|) + \sum_{i=1}^N \varepsilon_i |i\rangle\langle i|, \quad (1)$$

is a critical state [16]. In Eq. (1), $\langle ij \rangle$ denotes the sum over first neighbour sites.

We remind here below, the main points of procedure proposed by Sutherland [16]. The tiling is realized by fat and thin rhombuses, whose edges are labelled by either single or double arrows, as shown in Fig. 2(a).

The arrows of adjacent edges have to match to give rise to the Penrose tiling. The single arrows and the double arrows are considered as two vector fields, $\mathbf{A}$ and $\mathbf{B}$ whose circulation is zero. The divergence of such fields fixes the site energies ε_i at each site i as following

$$\varepsilon_i/t = \sum_{\langle j \rangle} \exp[-(r \nabla \cdot \mathbf{A} + s \nabla \cdot \mathbf{B})], \quad (2)$$

where r and s are two real parameters, and the sum runs over all the j sites, next-neighbours of the i site. More simply Eq. (2) can be written

$$\varepsilon_i/t = N_{i \rightarrow} e^{-r} + N_{i \leftarrow} e^r + N_{i \rightarrow \leftarrow} e^{-s} + N_{i \leftarrow \rightarrow} e^s, \quad (3)$$

$N_{i \rightarrow}$, $N_{i \leftarrow}$, $N_{i \rightarrow \leftarrow}$ and $N_{i \leftarrow \rightarrow}$ being the numbers of the outgoing and

ingoing single and double arrows at site i .

By construction, if s is different from 0, the ground-state is a critical state (it decays algebraically) if the five-fold symmetry center of the lattice corresponds to 5 outgoing single arrows (see Fig. 2(b)) [16]. Indeed it is possible to demonstrate that, starting from the symmetry center of the lattice, and following any path leading away from the center, the difference between ingoing and outgoing double arrows grows logarithmically as a function of the distance from the center [16]. The consequence is that the amplitude of the ground state, at $E=0$, decreases as a function of the distance from the center as a power law. More in details, if $s < s_c$ with $s_c = 4 \ln(\lambda)$, $\lambda = (\sqrt{5} + 1)/2$ being the golden number, the square of the wave function is not integrable, while it becomes integrable for $s > s_c$.

In the following we will fix $r=s$, and the site energies will depend only on the total number of outgoing arrows $N_{i, \text{out}}$ and ingoing arrows $N_{i, \text{in}}$, namely

$$\varepsilon_i/t = N_{i, \text{out}} e^{-s} + N_{i, \text{in}} e^s. \quad (4)$$

3. Dynamics

In this paper we investigate the interplay between the Sutherland Penrose potential and the interactions in the expansion dynamics of a Bose-Einstein condensate. We consider not only the case analyzed by Sutherland with the five-fold symmetry center composed by 5 outgoing simple arrows (SPL $\uparrow$) (Fig. 2(b)), but also the other possible case with 5 ingoing double arrows (SPL $\downarrow$) (Fig. 2(c)). Remark that these two configurations correspond to a potential local minimum and a potential local maximum respectively. The equation of motion for the condensate wavefunction ψ_i can be written as a Discretized Non-Linear Schrödinger Equation

$$i\hbar \partial_t \psi_i = -t \sum_{\langle j \rangle} \psi_j + \varepsilon_i \psi_i + g N |\psi_i|^2 \psi_i, \quad (5)$$

where g is the interaction strength, N the number of atoms in the condensate, and the wavefunction is normalized to 1, $\sum_i |\psi_i|^2 = 1$. We solve Eq. (5) using a second-order Runge-Kutta method. We consider a lattice of 416 sites, that is built by inflating a basic rhombus tile, and then copying it 5 times after having rotated it out of $2\pi/5$, $4\pi/5$, $6\pi/5$ and $8\pi/5$ radians. The lattice radius is $20d$, d being the rhombuses side length.

3.1. Dynamics in the absence of the interactions

First we study the time evolution of a Gaussian wavepacket in the absence of the interactions and as a function of s . In Figs. 3 and 4 we plot the width time-evolution of the wavepacket, that we define as

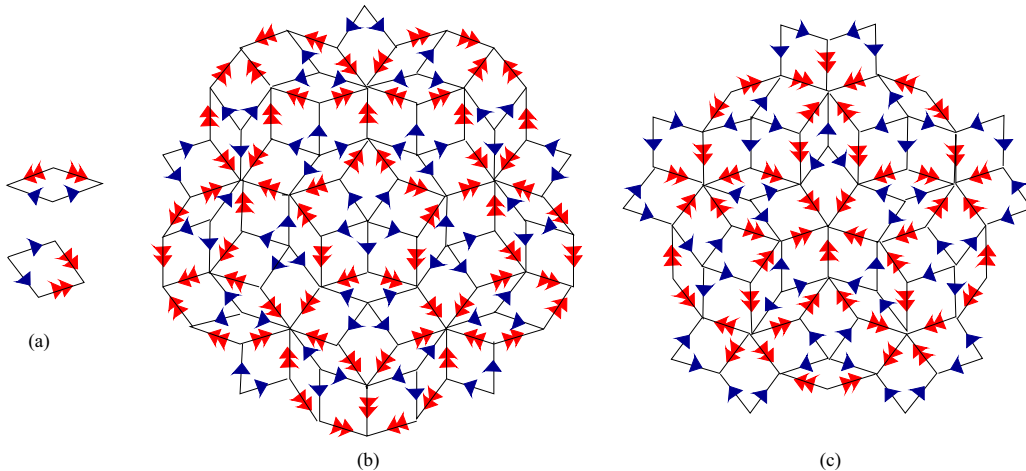


Fig. 2. Rhombus tiles (a) and Penrose tiling construction via arrows matching. Two types of centers are possible: (b) with 5 outgoing simple arrows, (c) with 5 ingoing double arrows.

Download English Version:

<https://daneshyari.com/en/article/5449972>

Download Persian Version:

<https://daneshyari.com/article/5449972>

[Daneshyari.com](https://daneshyari.com)