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Asymptotic nonlocal elasticity theory for the buckling analysis of embedded single-layered nanoplates/graphene sheets under biaxial compression

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ABSTRACT

A three-dimensional (3D) asymptotic formulation is developed for the buckling analysis of simply-supported, single-layered nanoplates/graphene sheets (SLNP and SLGS) embedded in an elastic medium and under biaxial compressive loads. In the formulation, the Eringen nonlocal elasticity theory is used to capture the small length scale effect, and the interaction between the SLNP/SLGS and its surrounding medium is simulated using a Pasternak-type foundation. After performing the mathematical processes of nondimensionalization, asymptotic expansion and successive integration, we finally obtain recursive sets of governing equations for various order problems. The nonlocal classical plate theory (CPT) is derived as a first-order approximation of the 3D nonlocal elasticity theory, and the governing equations for higher-order problems retain the same differential operators as those of nonlocal CPT, although with different nonhomogeneous terms. Some accurate nonlocal elasticity solutions of the critical load parameters of simply-supported, biaxially-loaded SLNP/SLGS with and without being embedded in the elastic medium are given to demonstrate the performance of the 3D asymptotic nonlocal elasticity theory.

1. Introduction

In recent years a number of nanostructured elements, such as the beam-like or circular hollow cylinder-like carbon nanotubes (CNT) [1] and plate-like graphene sheets (GS) [2], have been discovered. Due to their excellent mechanical, chemical, thermal and electrical material properties, CNT and GS have thus been introduced in a variety of potential applications to both micro- and nano-electro-mechanical systems [3–5]. The mechanical analyses of these nanoscale structures with and without being embedded in an elastic medium have therefore attracted considerable attention in order to extend their lifetimes and enhance their performance. This paper will focus on a literature survey related to the buckling analysis of single-layered nanoplates (SLNP) and single-layered GSs (SLGS) under biaxial compressive loads.

Since nonlocal continuum mechanics [6–8] is more computationally efficient than the atomistic [9] and hybrid atomistic-continuum mechanics approaches [10], most of the studies in the open literature with regard to the current issue are based on two-dimensional (2D) nonlocal plate theories, such as the nonlocal classic plate theory (CPT), first- and higher-order shear deformation plate theories (FSDPT and HSDPT), two-variable refined plate theory (TVRPT) and sinusoidal shear deformation plate theory (SSDPT), which are reformulated by combining their local counterparts with the Eringen nonlocal elasticity theory (ENET). Pradhan and Murmu [11,12], Murmu and Pradhan [13] and Pradhan and Phadikar [14] reformulated the local CPT to develop a nonlocal CPT for the buckling analysis of SLGS under uni- and bi-axial compression, in which the effects of the small length scale on the buckling characteristics of SLGS were closely examined. This nonlocal CPT was also used with the interatomic potential, which can accurately evaluate Young's modulus of the SLGS, in order to obtain an explicit formula for the critical load parameters of the SLGS, in works by Ansari et al. [15] and Ansari and Rouhi [16]. By means of the ENET, Aghababaei and Reddy [17] reformulated the local HSDPT [18,19] to carry out the bending and vibration analyses of nanoplates, in which the effects of the small length scale on the analytical solutions of stress, deformation and natural frequencies of the nanoplate were discussed. In conjunction with the von Karman geometrical nonlinearity and ENET, Reddy [20] developed the nonlinear formulations for bending of the nonlocal CPT, FSDPT and HSDPT of SLNP. Narendar [21] and Narendar and Gopalakrishnan [22] developed a nonlocal TVRPT to examine the influence of the small length scale on the buckling characteristics of micro- and nano-scale plates under biaxial compression. Thai et al. [23] reformulated the local SSDPT [24] to propose the nonlocal SSDPT for various mechanical analyses of micro- and nano-scale plates. In Thai et al., sinusoidal variations of transverse shear deformation through the thickness direction were considered, the shear

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correction factor was not required, and the Navier solutions for bending, buckling and free vibration of simply-supported, micro- and nano-scale plates were presented. Based on an isogeometric model, Ansari and Norouzzadeh [25] studied the nonlocal and surface effects on the buckling behavior of functionally graded material (FGM) nanoplates, in which the Mori-Tanaka scheme [26] was used to evaluate the through-thickness distributions of the effective material properties of the FGM nanoplate. An alternative approach to nonlocal continuum mechanics, the modified couple-stress theory, was used by Li and Pan [27] and Guo et al. [28] for the analysis of FGM anisotropic composite micro- and nano-scale plates and FGM piezoelectric ones.

A close literature survey shows that there are relatively few articles that carry out the three-dimensional (3D) buckling analysis of simply-supported, nanoplates and GS, as compared to the 2D analysis of these structures. To the best of the authors' knowledge, the perturbation method [29] has never been applied to the 3D structural analysis of nanoplates and GSs, even though it has been successfully applied to that of macrostructures, such as laminated composite structures [30–33] and functionally graded elastic/piezoelectric ones [34–36]. Within the 3D nonlocal elasticity theory combined with the full nonlinear kinematic terms, we thus developed an asymptotic theory for the 3D buckling analysis of simply-supported, SLNP/SLGS embedded in an elastic medium by using the perturbation method. In the formulation, we first reduce the fifteen partially differential equations (PDEs) of the 3D nonlocal elasticity theory to six PDEs in terms of six primary variables, which are three displacement components and three transverse shear and normal stress ones. By asymptotically expanding the primary variables and critical load parameters as a power series of a small geometric parameter, we finally obtain recursive sets of nonlocal governing equations for various order problems. The nonlocal CPT is derived as a first-order approximation of the 3D nonlocal elasticity theory, and the nonlocal governing equations for higher-order problems retain the same differential operators as those of the nonlocal CPT, although with different nonhomogeneous terms. We can obtain the Navier solutions of the leading-order problem by using the double Fourier series expansion method. By satisfying the solvability and normality conditions, the secular terms of higher-order problems can be removed, and the unique modal field variables can be obtained. The higher-order modifications can then be determined in a systematic manner. The effects of the small length scale, aspect ratio, Winkler stiffness and shear modulus of the medium on the critical parameters and their associated modal field variables for the SLNP and SLGS are also examined.

2. Basic equations of 3D nonlocal elasticity theory

In this work, a simply-supported, SLGS embedded in an elastic medium and under biaxial compressive loads is considered. The interaction between the SLGS and its surrounding medium is simulated using a Pasternak-type foundation. The relevant schematic diagrams for the biaxially-loaded SLGS and its corresponding nanoplate model are given in Fig. 1(a)–(c), in which the in-plane dimensions of the SLGS are L_xxL_y, and the effective total and one-half thickness of the GS are H and h, respectively, while $H = 2h$. A set of global Cartesian coordinates (x, y and z) is located at the mid-plane of the SLGS.

The main difference between the local and nonlocal elasticity theories is in their descriptions of the constitutive relation of a Hookean solid, rather than its related stress equilibrium equations. In the former, the stress components induced at a particular material point of the deformed elastic body depend only on the strain components induced at that point, while in the latter, these will depend on the strain components induced at all the material points of the continuum, due to the small length scale effect. According to Eringen [6,7] and Eringen and Edelen [8], the nonlocal constitutive behavior of an elastic body can be written as

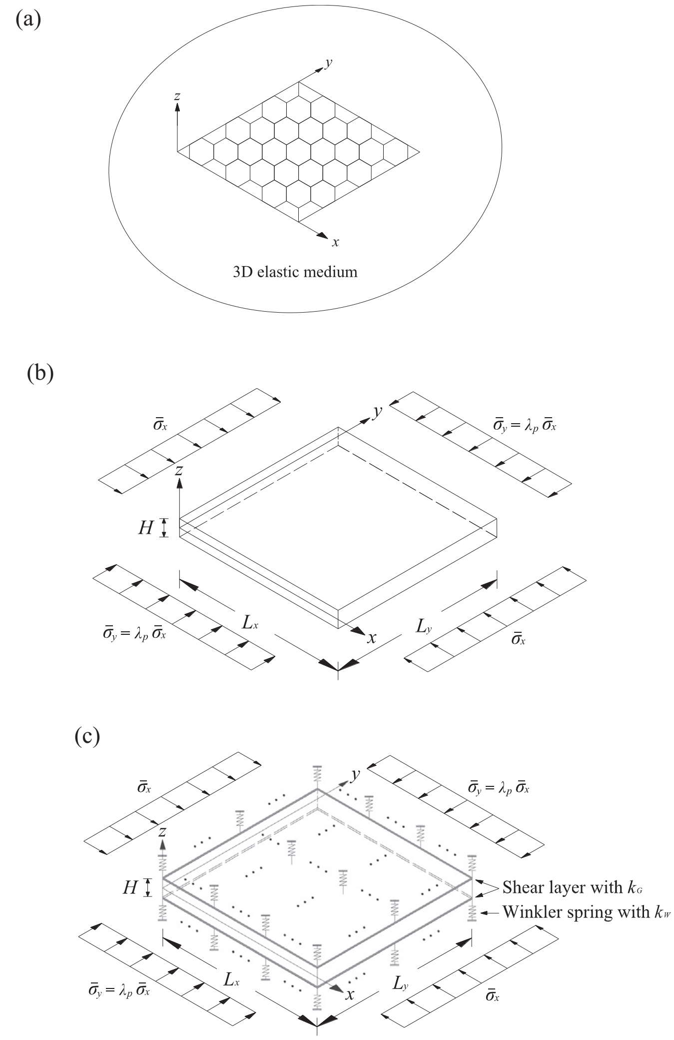

Fig. 1. (a) A single-layered GS embedded in an elastic medium, (b) a nanoplate model used for the analysis of the single-layered, bi-axially loaded and isolated GS, and (c) a nanoplate model used for the analysis of the single-layered, bi-axially loaded and embedded GS.

$$\sigma_{ij}(\mathbf{x}) = \int \alpha(\|\mathbf{x}-\mathbf{x}'\|, \tau)\ C_{ijkl}\ \varepsilon_{kl}(\mathbf{x}')dV(\mathbf{x}')\ , \qquad \forall\ \mathbf{x} \in V, \tag{1}$$

in which C_{ijkl} is the elastic modulus tensor of classical isotropic elasticity, and σ_{ij} and ε_{kl} are stress and strain tensors, respectively. $\alpha(\|\mathbf{x}-\mathbf{x}'\|, \tau)$ denotes the nonlocal modulus or attenuation function, which incorporates the constitutive equations into the nonlocal effect at the reference point $\mathbf{x}$ produced by local strain at the source $\mathbf{x}'$, and $\|\mathbf{x}-\mathbf{x}'\|$ is the Euclidean distance. $\tau = e_0\ a_0/l_0$, in which e_0 is a constant appropriate to each material, a_0 is an internal characteristic length (e.g., length of C–C bond, lattice parameter, or granular distance), and l_0 is an external characteristic length (e.g., crack length or wavelength). The value of e_0 needs to be determined from experiments or by matching the dispersion curves of plane waves with those of atomic lattice dynamics.

The integral-partial differential equations of Eq. (1) can be further reduced to singular partial differential equations of a special class of physically admissible kernels, as follows:

$$(1 - \mu\ \nabla^2)\ \sigma_{ij} = C_{ijkl}\ \varepsilon_{kl}, \tag{2}$$

where μ is the nonlocal parameter, and $\mu = (e_0\ a_0)^2$. ∇^2 is the Laplacian operator, in which $\nabla^2 = (\partial_{xx} + \partial_{yy} + \partial_{zz})$ is used for a 3D nonlocal elastic problem, while $\nabla^2 = (\partial_{xx} + \partial_{yy})$ and $\nabla^2 = \partial_{xx}$ for the 2D and 1D nonlocal elastic ones.