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Interaction of two-dimensional magnetoexcitons

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ABSTRACT

We study interaction of the two-dimensional magnetoexcitons with in-plane wave vector $\vec{k}_{\parallel}=0$, taking into account the influence of the excited Landau levels (ELLs) and of the external electric field perpendicular to the surface of the quantum well and parallel to the external magnetic field. It is shown that the account of the ELLs gives rise to the repulsion between the spinless magnetoexcitons with $\vec{k}_{\parallel}=0$ in the Fock approximation, with the interaction constant g decreasing inverse proportional to the magnetic field strength B ($g(0)\sim 1/B$). In the presence of the perpendicular electric field the Rashba spin-orbit coupling (RSOC), Zeeman splitting (ZS) and nonparabolicity of the heavy-hole dispersion law affect the Landau quantization of the electrons and holes. They move along the new cyclotron orbits, change their Coulomb interactions and cause the interaction between 2D magnetoexcitons with $\vec{k}_{\parallel}=0$. The changes of the Coulomb interactions caused by the electrons and by the holes moving with new cyclotron orbits are characterized by some coefficients, which in the absence of the electric field turn to be unity. The differences between these coefficients of the electron-hole pairs forming the magnetoexcitons determine their affinities to the interactions. The interactions between the homogeneous, semihomogeneous and heterogeneous magnetoexcitons forming the symmetric states with the same signs of their affinities are attractive whereas in the case of different sign affinities are repulsive. In the heterogeneous asymmetric states the interactions have opposite signs in comparison with the symmetric states. In all these cases the interaction constant g have the dependence $g(0)\sim 1/\sqrt{B}$.

1. Introduction

Magnetoexcitons are the bound states of the electron-hole pairs situated on their Landau levels and with the center of mass 2D wave vector $\vec{k}_{\parallel}$. The interaction between the two-dimensional (2D) magnetoexcitons differs essentially from the Wannier-Mott excitons. The Landau quantization of the 2D electrons and holes in the perpendicular magnetic field is characterized by the quantum orbits, whose radii do not depend on their effective masses, but only on the magnetic length l_0. In the Landau gauge the quantum orbits are characterized by the gyration points depending on the unidimensional wave numbers p and q side by side with the quantum numbers n_e and n_h of the electron and hole Landau levels. If $\vec{k}_{\parallel}=0$ and $n_e=n_h=0$, then the magnetoexcitons quantum orbits are overposed. If there is no an external electric field and the excited Landau levels are not taken into account the magnetoexcitons look as the neutral objects without the Coulomb interaction between them. It has been shown first in Refs. [1–3] that such magnetoexcitons form an ideal 2D Bose gas, and later studied in more detail in Ref. [4]. However, interaction between the magnetoexcitons with $\vec{k}_{\parallel}=0$ and with the electrons and holes being on the lowest Landau levels (LLLs) with $n_e=n_h=0$ become possible if the virtual transitions from the LLLs to the ELLs with arbitrary quantum numbers n and m and their return back is considered for the spinless electrons and holes taking part in the Coulomb scattering. In the second order of the perturbation theory these virtual transitions give rise to the indirect attraction between the particles supplementary to their direct Coulomb interaction. The impact of this supplementary interaction on the chemical potential of the Bose-Einstein condensed magnetoexcitons and on the ground sate energy of the metallic-type electron-hole liquid (EHL) was investigated in Ref. [5]. It was shown that the supplementary interaction is attractive in the Hartree approximation, and leads to repulsion in the Fock approximation stabilizing the Bose-Einstein condensed magnetoexcitons with $\vec{k}_{\parallel}=0$ against the collapse. One more possibility is the influence of the external electric field perpendicular to the surface of the quantum well and parallel to the external magnetic field. In this case the interaction does appear. The spin wave functions of the electrons and holes are characterized by the different Landau quantization numbers for different spin-projections and in this case the Rashba spin-orbit coupling (RSOC) comes into play. We will consider this variant starting with the exact solutions describing the Landau

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quantization of electrons and holes with spins projections under the influence of RSOC, the Zeeman splitting effects and the nonparabolicity (NP) of the heavy-hole dispersion law obtained recently in Refs. [6,7]. The Hamiltonian of the Coulomb electron-electron interaction for these conditions was obtained in Refs. [8,9].

Side by side with our coplanar system in the literature widely is discussed the two-layer separated electron-hole system in which electron and hole are localized in separate wells [10–14]. In such systems the indirect excitons are formed. Due to the distance between the wells the indirect excitons both in ground state and in the excited states have electrical dipole moments. So, the indirect excitons interact as the parallel dipoles in the two-layer structures or as the oppositely oriented dipoles in the three-layer structures. They are completely different from the case which is discussed below.

2. Influence of the excited Landau levels

The Hamiltonian of the Coulomb electron-electron interaction for the 2D electrons and the heavy holes situated only on the LLLs has the form [1–4]

$$H_{Coul}^{LLL}=\frac{1}{2}\sum_{\vec{Q}}W(\vec{Q})\left[\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})-\hat{N}_e-\hat{N}_h\right],\ W(\vec{Q})=\frac{2\pi e^2}{\tilde{S}|\vec{Q}|\varepsilon_0}e^{-\frac{Q^2l_0^2}{2}},\tag{1}$$

where $\tilde{S}$ is the quantum well surface area, ε_0 is the dielectric constant, $\hat{\rho}(\vec{Q})$ is the optical plasmon operator, which can be expressed through the electron and hole plasmon operators $\hat{\rho}_e(\vec{Q})$ and $\hat{\rho}_h(\vec{Q})$

$$\begin{aligned}\hat{\rho}(\vec{Q})&=\hat{\rho}_e(\vec{Q})-\hat{\rho}_h(\vec{Q}),\ \hat{\rho}_e(\vec{Q})=\sum_t e^{iQ_ytl_0^2}a^\dagger_{t+\frac{Q_x}{2}}a_{t-\frac{Q_x}{2}},\ \hat{N}_e=\hat{\rho}_e(0)\\&=\sum_t a_t^\dagger a_t,\ \hat{\rho}_h(\vec{Q})=\sum_t e^{-iQ_ytl_0^2}b^\dagger_{t+\frac{Q_x}{2}}b_{t-\frac{Q_x}{2}},\ \hat{N}_h=\hat{\rho}_h(0)=\sum_t b_t^\dagger b_t.\end{aligned}\tag{2}$$

Here $a_t^\dagger, a_t$ and $b_t^\dagger, b_t$ are the Fermi electron and the heavy-hole operators describing the particles on their LLLs with $n_e=n_h=0$. The supplementary interactions of the electrons and holes lying on the LLLs but undergoing the virtual Coulomb scattering with transitions to the ELLs and back are described by the Hamiltonian obtained in Ref. [5]:

$$\begin{aligned}H_{\sup}&=H_{\sup}^{e-e}+H_{\sup}^{h-h}+H_{\sup}^{e-h}=-\frac{1}{2}\sum_{p,q,k}\phi_{e-e}(p,q,k)a_p^\dagger a_q^\dagger a_{q+k}a_{p-k}\\&-\frac{1}{2}\sum_{p,q,k}\phi_{h-h}(p,q,k)b_p^\dagger b_q^\dagger b_{q+k}b_{p-k}-\sum_{p,q,k}\phi_{e-h}(p,q,k)a_p^\dagger b_q^\dagger b_{q+k}a_{p-k},\end{aligned}\tag{3}$$

where the coefficients $\phi_{i-j}(p,q,k)$ are determined by the formulas (14) of the Ref. [5]

$$\begin{aligned}\phi_{i-j}(p,q,z)&=\sum_{n,m}\frac{\phi_{i-j}(p,q,z;n,m)}{n\hbar\omega_{ci}+m\hbar\omega_{cj}},\ i,j=e,h,\ \phi_{i-j}(p,q,z;n,m)\\&=\sum_t F_{i-j}(p,0;q,0;p-t,n;q+t,m)\\&\times F_{i-j}(p-t,n;q+t,m;p-z,0;q+z,0).\end{aligned}\tag{4}$$

The Coulomb matrix elements are determined by the expressions (5) of the Ref. [5]

$$\begin{aligned}F_{i-j}(p,n;q,m;p-s,n';q+s,m')&=\iint d\vec{\rho}_1d\vec{\rho}_2\psi^{i*}_{n,p}(\vec{\rho}_1)\psi^{j*}_{m,q}(\vec{\rho}_2)\\&\frac{e^2}{\varepsilon_0|\vec{\rho}_1-\vec{\rho}_2|}\times\psi^i_{n',p-s}(\vec{\rho}_1)\psi^j_{m',q+s}(\vec{\rho}_2),\ i,j\\&=e,h,\end{aligned}\tag{5}$$

where $\psi^i_{n,p}(\vec{\rho}_1)$, $\psi^j_{m,q}(\vec{\rho}_2)$ are the 2D spinless electron and hole wave functions, whose Landau quantization states are given by quantum numbers n, m, and p, q in the Landau gauge. The electron-electron and hole-hole Coulomb integrals and the coefficients $\phi_{i-i}(p,q,z)$ depend on the difference $p-q$, whereas the electron-hole Coulomb integrals and the coefficients $\phi_{e-h}(p,q,z)$ depend on the sum $p+q$ of quantum numbers p and q. The Hamiltonian describing the full Coulomb interactions of the electron-hole system includes the direct and the supplementary components expressed by the formulas (1) and (3)

$$H_{Coul}=H_{Coul}^{LLL}+H_{\sup}.\tag{6}$$

The exciton creation operator and the wave function of the 2D magnetoexciton with wave vector $\vec{k}_\parallel$ obtained in Refs. [1–4] are

$$\begin{aligned}\psi^\dagger_{ex}(0,0;\vec{k}_\parallel)&=\frac{1}{\sqrt{N}}\sum_t e^{ik_ytl_0^2}a^\dagger_{t+\frac{k_x}{2}}b^\dagger_{-t+\frac{k_x}{2}},\\|\psi_{ex}(0,0;\vec{k}_\parallel)>&=\frac{1}{\sqrt{N}}\sum_t e^{ik_ytl_0^2}a^\dagger_{t+\frac{k_x}{2}}b^\dagger_{-t+\frac{k_x}{2}}|0>,\\N=\frac{\tilde{S}}{2\pi l_0^2},\ l_0^2&=\frac{\hbar c}{eB},\ \vec{k}_\parallel=\vec{a}_1k_x+\vec{a}_2k_y,\ <\psi_{ex}|\psi_{ex}>=1.\end{aligned}\tag{7}$$

Here $|0>$ is the vacuum state of the e-h system, N determines the degeneracy of the Landau levels, $\vec{a}_1$ and $\vec{a}_2$ are the unit in-plane vectors, l_0 is the magnetic length, and B is the magnetic field strength. The indices $n_e=n_h=0$ at the electron and hole operators were omitted.

The binding energy of the magnetoexciton determined by the Coulomb electron-hole direct and supplementary interactions is determined by the average value of the Hamiltonian (6)

$$\begin{aligned}&E_{ex}(0,0;\vec{k}_\parallel)=<\psi_{ex}(0,0;\vec{k}_\parallel)|H_{Coul}^{LLL}+H_{\sup}|\psi_{ex}(0,0;\vec{k}_\parallel)>,\\&<\psi_{ex}(0,0;\vec{k}_\parallel)|H_{Coul}^{LLL}|\psi_{ex}(0,0;\vec{k}_\parallel)>=-\sum_{\vec{Q}}W(\vec{Q})+2\sum_{\vec{Q}}W(\vec{Q})\\&\qquad\sin^2\left(\frac{[\vec{k}\times\vec{Q}]_zl_0^2}{2}\right)=-I_{ex}^{(0,0)}(\vec{k}_\parallel),\\&\sum_{\vec{Q}}W(\vec{Q})=I_l=\frac{e^2}{\varepsilon_0l_0}\sqrt{\frac{\pi}{2}},\\&E(\vec{k}_\parallel)=2\sum_{\vec{Q}}W(\vec{Q})\sin^2\left(\frac{[\vec{k}_\parallel\times\vec{Q}]_zl_0^2}{2}\right),\\&I_{ex}^{(0,0)}(\vec{k}_\parallel)=I_l-E(\vec{k}_\parallel),\ \lim_{k_\parallel\to\infty}E(\vec{k}_\parallel)=I_l,\\&<\psi_{ex}(0,0;\vec{k}_\parallel)|H_{\sup}|\psi_{ex}(0,0;\vec{k}_\parallel)>=-\frac{1}{N}\sum_{p,z}\phi_{e-h}(p,k_x-p;z)\\&\qquad e^{-ik_yzl_0^2}==-\sum_z\phi_{e-h}(p,k_x-p;z)e^{-ik_yzl_0^2}=-\Delta(\vec{k}_\parallel).\end{aligned}\tag{8}$$

In the last summation we take into account that $\phi_{e-h}(p,q,z)$ depends on the sum $p+q$ and $\phi_{e-h}(p,k_x-p,z)$ does not depend on p. The value of $\Delta(0)$ was determined in Ref. [5]

$$\Delta(0)=\frac{4I_l^2}{\pi(\hbar\omega_{ce}+\hbar\omega_{ch})}\cdot0.344.\tag{9}$$

It leads to the lowering of the lowest magnetoexciton energy level on the energy scale in comparison with its position in the absence of the supplementary indirect interaction and to the increasing of its ionization potential.

The following wave function $|\phi_0>$ is used to determine the interaction of two magnetoexcitons with electrons and holes being on the LLLs with the wave vectors $\vec{k}_\parallel=0$

$$\begin{aligned}&|\phi_0>=\hat{\psi}^\dagger_{ex}(0,0;0)\hat{\psi}^\dagger_{ex}(0,0;0)|0>=\frac{1}{N}\sum_{st}a_t^\dagger a_s^\dagger b_{-s}^\dagger b_{-t}^\dagger|0>,\\&<\phi_0|\phi_0>=2\left(1-\frac{1}{N}\right).\end{aligned}\tag{10}$$

Our goal is to determine the interaction between the magnetoexcitons.