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Oblique transport of gyrotactic microorganisms and bioconvection nanoparticles with convective mass flux

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ABSTRACT

The present study deals with examination of steady two dimensional nanofluid containing both nanoparticles and gyrotactic microorganisms. Moreover the study comprises stagnation point flow of an obliquely striking nanofluid. The governing partial differential equations are complex and highly non-linear in nature. These are converted into system of ordinary differential equations using suitable transformations. The system is then solved numerically using shooting technique with Runge - Kutta Fehlberg method of order 5. Further, effect of different physical parameters on velocity $f'(\eta)$, temperature $\theta(\eta)$, density of motile microorganisms $w(\eta)$ and concentration $\phi(\eta)$ along with skin friction coefficient C_f, local Nusselt Nu_x, Sherwood Sh_x and density of motile microorganism Nn_x numbers have been discussed through graphs and tables. Results depict that temperature, concentration, density of motile microorganisms and local Nusselt number are increasing functions of thermophoresis parameter N_t. Whereas N_t contributes in lessening Sherwood and local density numbers.

1. Introduction

Increasing attention on flow and heat transfer in boundary layer induced by continuously moving and stretching surface has gained importance because of many industrial and technological applications. These applications include manufacturing filaments, polymer sheet and wind up roller etc. The pioneer work on boundary layer flow over flat surface was initiated by Sakiadis [1]. The analysis of heat and mass transfer towards stretching sheet induced by suction and blowing was investigated by Gupta and Gupta [2]. Takhar et al. [3] investigated boundary layer flow induced by stretching surface. The contribution of heat and mass transfer on MHD flow toward a shrinking sheet with suction was studied by Hashim et al. [4]. Moreover concept of stretching sheet for Newtonian as well as non Newtonian fluid flow has been broadly investigated [5–7].

The movement of fluid near stagnation region of a solid surface either fixed or moving is described by stagnation point flow. Stagnation point flow has also received considerable attention of the researchers due to its applications in aerospace technology and engineering. Flow over the tips of submarines, oilships, rockets and aircrafts are few examples of stagnation point flows. Hiemenz [8] initially studied 2D stagnation point flow induced by fixed semi-infinite wall and used similarity transformation. He reduced Navier - Stokes equations and transformed them to ordinary differential equations. This problem was extended to heat transfer analysis on stagnation point flows over stretching surface by Mahapatra and Gupta [9,10]. Hayat et al. [11] examined stagnation point flow of second grade fluid over unsteady stretching surface in presence of variable free stream. Numerical investigation on stagnation point flow induced by stretching surface is worked out by Nawaz et al. [12]. They inspected influence of thermal and Newtonian heating in flow. Stagnation point flow generates either by fluid striking the surface orthogonally or at an arbitrary incidence angle. Recently, non orthogonal stagnation point flow over stretching sheet was discussed by Labropulu et al. [13] Nadeem et al. [14] investigated non-orthogonal stagnation point flow of a nanofluid over stretching sheet. Few notable articles in this regards are [15,16].

As rapid development of human society, energy crises arise day by day. This challenge is coped by use of solar energy which has been regarded one of the best source of energy renewal via slightest environmental impact. Potential efficiency is improved by using nanofluid as a working fluid. The study of heat and mass transfer in nanofluids has been encountered in many industrial and engineering applications such as in power generation, heating and cooling processes etc. Choi [17] found that when small nanosized particles are suspended into base fluids, thermal conductivity of fluids can be enhanced. The behavior of nanoparticles can be checked through Brownian motion and thermophoresis effects when turbulent effects are absent. This is examined by Buongiorno [18]. He established conservation equations on these effects and this work gained importance in field of heat transfer. Nanofluid flow near a stagnation-point

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towards a stretching surface is reported by Mustafa et al. [19]. They found that temperature is decreasing function of Brownian motion and thermophoresis parameters. Zaimi et al. [20] investigated boundary layer flow of nanofluid towards a nonlinearly permeable stretching/shrinking surface. They explained special effects of thermophoresis and Brownian motion parameters and concluded that nanoparticles concentrations have opposite effects for thermophoresis and Brownian motion parameters.

Nanofluid containing microorganism is an interesting topic of research now a days. The collective motion of motile microorganism within the fluid produce density gradient generating macroscopic convection termed as bioconvection. Origin of bioconvection is like natural convection where unbalanced thickness stratification is incorporated. Many aspects of bioconvection problems in suspension that contain solid particles are investigated by several researchers. Few articles explaining the bioconvection of motile microorganisms are [21,22]. Kuznetsov and Avramenko [23] made the first investigation on bioconvection in suspensions having small solid particles and gyrotactic microorganisms. Boundary layer flow of nanofluid containing gyrotactic microorganisms towards flat plate induced by porous medium and stagnation point flow over stretching/shrinking sheet are studied by Aziz et al. [24] and Zaimi et al. [25]. Recently, Das et al. [26] studied nanofluid bioconvection in presence of gyrotactic microorganisms and chemical reaction in a porous medium. Multiple slip effects on bioconvection of nanofluid flow containing gyrotactic microorganisms and nanoparticles was examined by Sk et al [27]. They concluded that the growth in nanoparticle density slip parameter consequences 23.64% shrinkage in mass transfer proportion and 10.11% decline in microorganism density number and alternatively heat movement speed lessened by 11.16% because of temperature slip effect in the stream. Few useful articles are cited for reader's interest see ref. [28–30].

In view of above discussed scenarios, present analysis is an attempt to investigate oblique stagnation point flow of nanofluid contains gyrotactic microorganisms over a stretching sheet. The main emphasis is given to study the relation between nanoparticles characteristics and density of motile microorganisms. This is the first attempt to consider oblique transfer of gyrotactic microorganism suspended nanofluid in presence of convective mass flux condition.

2. Mathematical development and governing model

Consider a steady, incompressible, two dimensional bidirectional oblique stagnation point flow of a nanofluid over an elongating stretching surface at temperature T_w. The surface is considered to be along the x-axis ($y = 0$). The density N_w of gyrotactic microorganism is assumed to be blended within nanofluid at the surface. Moreover, flow phenomenon is discussed under the effect of mass flux condition at the surface. Far from the surface, constant ambient temperature T_∞, ambient concentration C_∞ and zero density of motile microorganism is considered. It is further assumed that flow of fluid interrupts on stretching surface at an arbitrary angle of incidence γ with velocity $\bar{v}_e(\bar{u}_e, \bar{v}_e)$ as demonstrated in following physical diagram.

Under these assumptions governing equations are

$$\nabla . \bar{V} = 0, \tag{1}$$

$$\rho_f(\bar{V}.\nabla)\bar{V} = -\nabla p + \mu\nabla^2\bar{V} + gC\rho_p + g(1-C)\{\rho_f - \rho_f\beta(T - T_\infty)\} + m\Delta\rho N g, \tag{2}$$

$$\bar{V}.\nabla T = \alpha\nabla^2 T + \tau\left[D_B\nabla C.\nabla T + \left(\frac{D_T}{T_\infty}\right)\nabla T.\nabla T\right], \tag{3}$$

$$\bar{V}.\nabla C = D_B\nabla^2 C + \left[\left(\frac{D_T}{T_\infty}\right)\nabla^2 T\right], \tag{4}$$

$$\nabla . \bar{j} = 0, \tag{5}$$

where $\bar{V}$ is fluid velocity vector, p is pressure, T is fluid temperature, C is nanoparticle concentration, N is concentration of microorganisms, $\bar{j}$ is microorganisms flux, ρ_f is density of base fluid, α is thermal diffusivity of nanofluid, μ stands for fluid viscosity, β represents volume expansion coefficient, ρ_p is density of nanoparticles, m is average volume of microorganism, D_B Brownian diffusion coefficients, D_T thermophoretic diffusion coefficient, $\Delta\rho = \rho_{cell} - \rho_{bf}$ is difference between densities of cell and base fluid and g is gravitational acceleration. Microorganism flux j is given as [25].

$$\bar{j} = N(\bar{V} + \hat{V}) - D_n\nabla N, \tag{6}$$

where $\hat{V} = (bW_c/\Delta C)\nabla C$, D_n is diffusivity of microorganisms, b is chemotaxis constant and W_c is maximum cell swimming speed. Using Eqs. (1–5), by Implementing boundary layer approximation and eliminating pressure gradient then we achieved following governing equations

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0, \tag{7}$$

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} = \bar{\nu}\frac{\partial^2\bar{u}}{\partial\bar{y}^2} - \frac{g}{\rho_f}\left[\begin{array}{c}(\rho_p - \rho_\infty)(C - C_\infty) - \\ (1 - C_\infty)\beta\rho_{f_\infty}(T - T_\infty) - \gamma\nabla\rho N\end{array}\right], \tag{8}$$

$$\bar{u}\frac{\partial T}{\partial x} + \bar{v}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right\}, \tag{9}$$

$$\bar{u}\frac{\partial C}{\partial x} + \bar{v}\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{10}$$

$$\bar{u}\frac{\partial N}{\partial x} + \bar{v}\frac{\partial N}{\partial y} + \frac{\partial}{\partial y}(N\bar{v}) = D_n\left(\frac{\partial^2 N}{\partial y^2}\right), \tag{11}$$

corresponding boundary conditions are

$$\left.\begin{array}{l}\bar{u} = u_w(\bar{x}) = c\bar{x},\ v = 0,\ T = T_w, \\ D_B\frac{\partial C}{\partial\bar{y}} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial\bar{y}} = 0,\ N = N_w,\end{array}\right\}\text{at } y = 0,$$
$$\left.\begin{array}{l}\bar{u} \to \bar{u}_e(\bar{x}) = ax\sin(\gamma) + by\cos(\gamma), \\ v \to -ay\sin(\gamma),\ \ T \to T_\infty,\ \ C \to C_\infty,\ N \to 0\end{array}\right\}\text{as } y \to \infty. \tag{12}$$

where $\bar{u}$ and $\bar{v}$ are velocity components in x- and y- directions, respectively, a and c are positive constants, T_w is temperature of sheet and T_∞ is temperature of fluid far away from sheet and γ is a parameter (see [15]). The flow model depends on γ either it is favorable or not. If $0 < \gamma < \pi/2$ then flow is favorable and for unfavorable its range will be $\pi/2 < \gamma < \pi$. Applying following non-dimensional variables incorporating with similarity transformations [15].

$$x = \sqrt{\frac{c}{\nu}}\bar{x},\ \eta = \sqrt{\frac{c}{\nu}}\bar{y},\ \ \psi = \frac{\bar{\psi}}{\nu},\ \psi = xf(\eta) + g(\eta),\ u = xf'(\eta) + g'(\eta),$$
$$v = f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \ C(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\ \chi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}. \tag{13}$$

into Eqs. (7–12) we have

$$f''' + ff'' - f'^2 + C_1 = 0, \tag{14}$$

$$g''' + fg'' - f'g' + (G_r\theta - N_r\phi + R_b\chi) + C_2 = 0, \tag{15}$$

$$\theta'' + \mathrm{Pr}f\theta' + N_b(\theta'\phi') + N_t(\theta')^2 = 0, \tag{16}$$

$$\phi'' + L_e(f\phi') + \frac{N_t}{N_b}\theta'' = 0, \tag{17}$$

$$\chi'' + Sc(f\chi') - P_e(\phi'\chi' + \phi''\chi) = 0, \tag{18}$$

$$f(\eta) = 0,\ f'(\eta) = 1,\ g(\eta) = 0,\ g'(\eta) = 0,\ \theta(\eta) = 1,\ N_b\phi'(\eta) + N_t\theta'(\eta) = 0,$$
$$\chi(\eta) = 1,\ \text{at}\eta = 0,\ f'(\eta) \to \lambda\sin\gamma,\ g''(\eta) \to k\cos\gamma,\ \theta(\eta) \to 0,\ \phi(\eta) \to 0,\ \chi(\eta) \to 0 \text{ as } \eta \to \infty. \tag{19}$$