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Wave dispersion in viscoelastic single walled carbon nanotubes based on the nonlocal strain gradient Timoshenko beam model

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ABSTRACT

Based on the nonlocal strain gradient theory and Timoshenko beam model, the properties of wave propagation in a viscoelastic single-walled carbon nanotube (SWCNT) are investigated. The characteristic equations for flexural and shear waves in visco-SWCNTs are established. The influence of the tube size on the wave dispersion is clarified. For a low damping coefficient, threshold diameter for shear wave (SW) is observed, below which the phase velocity of SW is equal to zero, whilst flexural wave (FW) always exists. For a high damping coefficient, SW is absolutely constrained, and blocking diameter for FW is observed, above which the wave propagation is blocked. The effects of the wave number, nonlocal and strain gradient length scale parameters on the threshold and blocking diameters are discussed in detail.

1. Introduction

Owing to its low weight, high stiffness and large elastic strain, carbon nanotube (CNT) has become one of the most promising structural elements for nano-electronics, nano-devices and nano-composites [1–8]. Numerous efforts have been paid to clarify the mechanical performance of CNTs including bending, buckling, and free vibration [9–12]. Especially in recent years, with the growing interest in terahertz physics of nanoscale materials and devices, the studies of vibration and wave propagation in CNTs attract more and more attention [13–16]. Since the size of a CNT is close to the atom scale, the size effect becomes dominant. Consequently, instead of classical continuum theory, the nonlocal elastic theory [17,18], which characterizes the size effect by assuming that the stress at a reference point is a function of the strain field at all points in the body, is widely adopted in the analysis of wave propagation in CNTs for a better prediction of their mechanical performance.

Wang et al. [19] studied wave propagation in the viscous fluid-conveying SWCNTs based on the nonlocal Euler-Bernoulli beam theory. Lu et al. [20] adopted the nonlocal Timoshenko beam theory to investigate the wave properties of CNTs. Kiani [21] investigated the influences of the nonlocal effect and longitudinal magnetic field on the wave characteristics of SWCNTs by use of nonlocal Rayleigh, Timoshenko and higher-order beam theories, and found that the nonlocal effect should not be neglected in the wave dispersion analysis for both FW and SW. Recently, viscoelastic properties of CNTs have been reported for an operational temperature range of −196 °C to 1000 °C [22]. The new properties caused by the viscoelastic characteristic and its potential application have stirred new interests in the analysis of the dynamic performances of CNTs. Bahaadini and Hosseini [23] explored the effects of nonlocal parameter and slip condition on free vibration and flutter instability of viscoelastic cantilever CNTs conveying fluid. Ghorbanpour Arani *et al.* [24] investigated nonlinear free vibration of double bonded visco-CNTs conveying viscous fluid embedded by visco-Pasternak medium. Based on the nonlocal elastic theory and the Kelvin model, Pang et al. [25] investigated the viscoelastic damping and nonlocal effects on the frequency of FW in viscoelastic SWCNTs. These works have clearly showed that the viscoelastic damping and size effects play important roles on the dynamic performance of visco-CNTs. However, it was lately reported [26–31] that there are some limited issues in the capability of nonlocal elasticity theory for identifying size-depended stiffness. The nonlocal elasticity theory cannot well predict the stiffness enhancement effects, which are noticed from experimental measurements, as well as the strain gradient theory [32]. Moreover, it is doubted that there needs no extra non-classical boundary conditions in the nonlocal elasticity theory. Actually, some additional non-classical boundary conditions are expected to appear in higher-order elasticity models [26,33,34]. In order to address these limited issues, Lim et al. [29] developed the nonlocal strain gradient theory, introducing both the nonlocal parameter and the strain gradient length scale parameter into the constitutive equation. The nonlocal strain gradient theory consideres not only the non-gradient nonlocal elastic stress field [17,18], but also the nonlocality of higher-order strain gradients stress field [35]. It reason-

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ably explains the size-dependent wave propagation behavior of CNTs, and its prediction fits well with the molecular dynamics (MD) simulation results.

By using the nonlocal strain gradient theory and Rayleigh beam theory, Li et al. [30] investigated the influences of the magnetic field and surface effects on the FW dispersion in viscoelastic SWCNTs. The results showed that the wave dispersion relation can be significantly influenced by the nonlocal parameter and the strain gradient length scale parameter. Li and Hu [31] explored the effect of fluid flow on the propagation characteristics *of viscoelastic FW in viscoelastic SWCNTs*. Tang et al. [36] investigated the effect of tube size on the propagation characteristics of viscoelastic FW in viscoelastic SWCNTs by adopting nonlocal strain gradient Rayleigh beam model. The blocking diameter of FW is observed and clearly dependent on the damping coefficient, nonlocal and strain gradient length scale parameters. These works to some extend clarify the wave propagation and dispersion properties in visco-CNTs. Unfortunately, since the Rayleigh beam model is adopted, the effect of shear deformation, which has a significant influence on the dynamic performance of CNTs [37], has been ignored. Although Lei et al. [37] analyzed the vibration behavior of a nonlocal viscoelastic nanobeam based on the Timoshenko beam theory, when the effect of the shear deformation is considered, the viscoelastic properties of CNTs and size effects on the propagation characteristics of viscoelastic shear wave (SW) have not yet been fully considered.

The present paper is aiming to investigate the size effects on the propagation characteristics of viscoelastic FW and SW in a viscoelastic SWCNT based on the nonlocal strain gradient Timoshenko beam model. The distinct propagation phenomena of viscoelastic FW and SW related to the tube size is discussed in detail.

2. Atomic structure of a SWCNT

The atomic structure of a SWCNT is theoretically assumed to be formed by rolling up a graphite sheet along certain direction, which is defined by the chiral vector [38]

$$\mathbf{C}_h = n\mathbf{a}_1 + m\mathbf{a}_2 \tag{1}$$

where $\mathbf{a}_1$ and $\mathbf{a}_2$ are the two unit directional vectors. The chirality indices (n, m) decides the structure around the circumference (Fig. 1).

The diameter D of the carbon nanotube (n, m) is expressed as [38]

$$D = \frac{\sqrt{3}\,a}{\pi}\sqrt{n^2 + nm + m^2} \tag{2}$$

where a is the length of the carbon-carbon bond with the value a=0.142 nm.

Experimental studies have been applied to measure the elastic properties of CNTs [39–42]. Owing to the diversity in the reported experimental data, theoretical investigation and numerical simulations on the elastic properties of CNTs have been used to justify the experimental findings and offer some useful information. Using a transmission electron microscope, Poncharal et al. [42] found that multiwalled carbon nanotubes with smaller tube diameters had higher

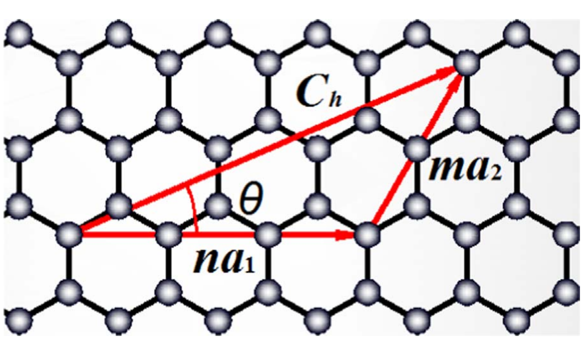

Fig. 1. Schematic representation of the chiral vector.

Young's modulus. Yao and Lordi [43] obtained the same results for the Young's modulus of SWCNTs by the use of MD simulation. Based on these experimental and MD simulation results, Sakharova et al. [44] evaluated the Young's and shear modulus of SWCNTs and established the relation between the material and geometrical parameters. They gave the approximated relations between the tube diameter and the tensile rigidity EA, bending rigidity EI, and torsional rigidity GJ, respectively, as

$$EA = \alpha(D - D_0) \tag{3}$$

$$EI = \beta(D - D_0)^3 \tag{4}$$

$$GJ = \gamma(D - D_0)^3 \tag{5}$$

where $\alpha, \beta, \gamma, D_0$ are the fitting parameters, D the tube diameter.

For CNTs, the cross-sectional area A and moments of inertia I and J are written as

$$A = \frac{\pi}{4}[(D + h)^2 - (D - h)^2] = \pi Dh \tag{6}$$

$$I = \frac{\pi}{64}[(D + h)^4 - (D - h)^4] \tag{7}$$

$$J = \frac{\pi}{32}[(D + h)^4 - (D - h)^4] \tag{8}$$

where h is the CNT wall thickness.

Combining Eqs. (3), (4), (6) and (7), the expression for tube diameter D is given as

$$D = \sqrt{8\frac{EI}{EA} - h^2} \tag{9}$$

Then, the Young's and shear modulus of SWCNTs are given as

$$E = \frac{EA}{A} = \frac{\alpha(D - D_0)}{\pi h\sqrt{8\frac{\beta(D-D_0)^2}{\alpha} - h^2}} \tag{10}$$

$$G = \frac{GJ}{J} = \frac{\alpha\gamma(D - D_0)}{2\pi h\beta\sqrt{8\frac{\beta(D-D_0)^2}{\alpha} - h^2}} \tag{11}$$

3. Wave propagation in SWCNTs

It was recently reported that both the nonlocal parameter and the strain gradient length scale parameter are necessary in describing the size-depended wave propagation behavior of CNTs [29–31]. Aiming to take into account both the effects of the nonlocal parameter and the strain length scale parameter, the nonlocal strain gradient theory [29] is adopted in our discussion. The nonlocal strain gradient constitutive relation for the normal stress and strain is expressed as [29]

$$\left[1 - (e_1a)^2\frac{\partial^2}{\partial x^2}\right]\left[1 - (e_0a)^2\frac{\partial^2}{\partial x^2}\right]\sigma_{xx} = E\left[1 - (e_1a)^2\frac{\partial^2}{\partial x^2}\right]\varepsilon_{xx} - El^2\left[1 - (e_0a)^2\frac{\partial^2}{\partial x^2}\right]\frac{\partial^2\varepsilon_{xx}}{\partial x^2} \tag{12a}$$

where x is the axial coordinate. σ_{xx} is the normal stress and ε_{xx} is the normal strain. e_0a is the nonlocal parameter introduced to consider the low-order elastic stress field with e_0 the nonlocal material constant. e_1a is introduced to describe the higher order nonlocal elastic stress field with e_1 is the related material constant. Many investigators have suggested a conservative estimate of the nonlocal parameter e_0a for CNTs that the value of the nonlocal parameter should be smaller than 2 nm [45–47]. l is the strain gradient length scale parameter introduced to determine the significance of higher-order strain gradient field. Some researches have been applied to determine the material length scale parameter for some materials [32,48,49]. Assuming that e=e_0=e_1 [29], we have