Solar Energy 141 (2017) 59–69

Contents lists available at ScienceDirect

Solar Energy

journal homepage: www.elsevier.com/locate/solener

A novel design concept for a reflector of parabolic trough concentrator based on pure bending and correction principle

Zhongzhu Qiu [a,*], Peng Li [b,*], Chunying Li [c], Qunzhi Zhu [a], Tao Zhang [a], Chengyao Wang [a]

[a] College of Energy and Mechanical Engineering, Shanghai University of Electric Power, China
[b] College of Mechanical Engineering, Tongji University, China
[c] College of Real Estate, Beijing Normal University, China

ARTICLE INFO

Article history:
Received 2 September 2016
Received in revised form 26 October 2016
Accepted 3 November 2016

Keywords:
Pure bending
Correction
Solar parabolic trough
Reflector
Design concept

ABSTRACT

It is cost effective to manufacture a reflective surface of a solar parabolic trough concentrator in a parabolic cylinder shape by pure bending of a flat sheet. However, the surface manufactured by such an elastic bucking is only a coarse approximation of a parabola, which is not precise enough for meeting the demand of a high quality concentrator. A novel design concept was proposed to remove this error by applying an external force on both edge of a sheet, together with two additional forces: the edge torsion force and the pressing force. The optimal position and magnitude of two additional forces were figured out among various conditions, thereby a theoretically maximal concentration ratio up to 115 was found. Prototypes and engineering realizations of this method were also presented. This method provides a very attractive approach for low cost and high efficiency solar energy solutions.

1. Introduction

A parabolic trough is a concentrating solar thermal energy collector that uses a mirror in a form of a parabolic cylinder to reflect solar rays towards a receiver tube located at the focus line of the parabola. It has the advantage of being reliable and of low cost, and it can reach a working temperature high enough for efficient power generation (Price et al., 2002; Fernández-García et al., 2010; Wirz et al., 2014; Tsai, 2016).

The geometric precision and manufacturing cost of the parabolic mirror are the fundamental factors in the production of a parabolic trough. Traditionally, the support of the mirror is a rigid sheet precisely preformed to the shape of a parabolic cylinder (Hatwaambo et al., 2008; Thomas and Guven, 1993). This constitutes an important part of the product cost, because both the rigidity of the material and the high precision requirement of the forming process are expensive.

Alternatively, it is possible to make use of the pure bending of a flat sheet to form an approximately parabolic cylinder, as under certain conditions, the shape of the buckled flat sheet is usefully close to the parabola (Li et al., 2011a,b; Li and Dubowsky, 2011). While the cost advantage of this approach is obvious, it only provides a rough approximation that does not meet the precision requirement of a high quality solar concentrator, nor does it allow any correction of eventual defects of the material or the manufacturing processes.

The aim of this article is to propose an improved approximation of the parabolic cylinder by the elastic deformation of a flat sheet, by applying additional external forces besides the buckling force. These involve in a torsion force at each edge of the sheet, and two pressing force in the curve. With optimal amount of the forces and optimal positions of the points, this method leads to an approximation of the parabola with a much higher precision.

2. The buckling curve

2.1. The curve equation

The buckling of a flat sheet can be seen as a simple development of a straight rod on a perpendicular dimension. Therefore it is enough to consider the hinged pure bending of a straight uniform rod, whose diameter is supposed to be infinitesimally small. Suppose that the original rod is under a horizontal position, and that its weight is infinitesimal, thus there is no effect of the gravity. When two edges of the rod are compressed to some extent by an external force, F, the rod will elastically deform to be a curved shape (see Fig. 1).

* Corresponding authors.
E-mail addresses: qiuzhongzhu@shiep.edu.cn (Z. Qiu), lipeng99@tongji.edu.cn (P. Li).

http://dx.doi.org/10.1016/j.solener.2016.11.008
0038-092X/

Nomenclature

a_i	factor, where $i = 1, 2, 3 \ldots$
A	area, m^2
b	width of a cross section, m
d	diameter, m
$d(x_r)$	focus deviation, m
$d_1(x)$	the upper bound of the reflected cone, m
$d_2(x)$	the lower bound of the reflected cone, m
$D(x)$	the distance between the reflecting point to the center of the receiver, m
E	Young's modulus of elasticity, N m^{-2}
F	the external force, N
h	height of a cross section, m
I	the inertia moment, m^{-4}
$k(x)$	the curvature of the curve at (x, y), m^{-1}
y'_0	$y'(x_0)$
k	The curvature caused by external force (F), m^{-1}
M	moment, N m
R_1	the incident solar radiation beam
R_2	the reflected solar radiation beam
W	the opening width of the curve: $W = x_1 - x_0$, m
x	horizontal coordinate of a point on a curve, m
y	vertical coordinate of a point on a curve, m
y', $y'(x)$	the firs derivative of the curve
y'', $y''(x)$	the second derivative of the curve
y_f	the position of the receiver on the y axis, m

Greek symbols

α	the angle between the vector $(x - x_p, y - y_p)$ and the tangent vector of C at (x_p, y_p), rad
θ	the tangent angle of the curve at a certain point, rad
θ_c	the top angle of the cone, rad
ρ	the concentration radio

Subscription

0	end point of the curve
c	cone
e	edge torsion force
f	focus point
h	horizontal component
v	vertical component
m	pressing force
r	reflection
rec	receiver tube
p	position at which torsion force applies
q	position at which pressing force applies
sys	overall

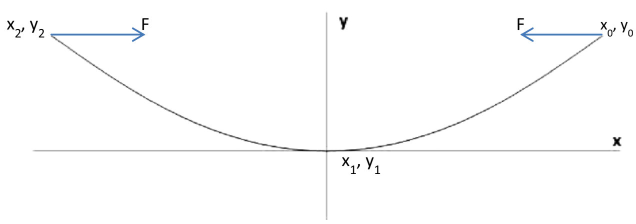

Fig. 1. The buckling curve.

The curvature of the curve at a point (x, y) is proportional to the moment of force exercised on that point in terms of the principle on pure bending, therefore for the right half part of the curve, the curvature can be given by Timoshenko and Gere (1961), Beer et al. (1972).

$$k(x) = \frac{M}{EI} \tag{1}$$

As the moment, M, is equal to $F(y_0 - y)$ Li et al., 2011a, substituting it into Eq. (1), the following equation was obtained

$$k(x) = -\frac{F}{EI}(y - y_0) \tag{2}$$

where $k(x)$ is the curvature of a curve at (x, y), and can be calculated by $k(x) = \frac{y''(x)}{(1+y'(x)^2)^{3/2}}$ (Yates, 1952). E is Young's modulus of elasticity. I is the inertia moment of the section area, and determined by the equation $I = \int_A y^2 dA$, for a rectangular beam, $I = \frac{bh^3}{12}$. F is an external force, denotes a scalar here.

Substituting Eq.(2) into $k(x) = \frac{y''(x)}{(1+y'(x)^2)^{3/2}}$, it yields:

$$y'' = -\frac{F}{EI}(y - y_0)(1 + y'^2)^{3/2} \tag{3}$$

Under the hypotheses, the curve is symmetric, that is, $y_2 = y_0$ and $x_0 = -x_2$. By symmetry, the computation needs only be done on the right half of the curve, for the interval $[x_1, x_0]$. And we can fix the first boundary conditions $y(0) = 0$ from the Fig.1 and the second boundary condition on $y'(0) = 0$ by symmetry, thus the boundary conditions are available as following:

$$\begin{cases} y(0) = 0 & (1) \\ y'(0) = 0 & (2) \end{cases} \tag{4}$$

However, it is necessary to determine y_0 to solve Eq. (3). Actually $(1 + y'^2)^{3/2}$ can be written in the form of series as below

$$(1 + y'^2)^{3/2} = 1 + \frac{3}{2}y'^2 + \frac{3}{8}y'^4 + \ldots \tag{5}$$

Substitute Eq (5) into Eq. (3), yielding

$$y'' - \frac{F}{EI}(y - y_0)\left(1 + \frac{3}{2}y'^2 + \frac{3}{8}y'^4 + \ldots\right) = 0 \tag{6}$$

Based on the boundary condition y(0) = 0 and y′(0) = 0, at x = 0 the function y, in terms of Tailor formula, can be expressed as

$$y = a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + \ldots \tag{7}$$

From Eq. (7) the following two equations can be obtained:

$$y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 + \ldots \tag{8}$$

$$y'' = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + 30a_6x^4 + \ldots \tag{9}$$

where a_1 is equal to 0 due to y′(0) = 0. To substitute Eqs. (7)–(9) into Eq. (6), the following equation was obtained

$$\begin{aligned} &2a_2 - \frac{F}{EI}y_0 + 6a_3x + \left[12a_4 - \frac{F}{EI}(6y_0a_2^2 - a_2)\right]x^2 \\ &+ \left[20a_5 - \frac{F}{EI}(18y_0a_2^2 - a_3)\right]x^3 \\ &+ \left[30a_6 - \frac{F}{EI}\left(6y_0a_2^4 + 24y_0a_2a_4 + \frac{27}{2}y_0a_3^2 - 6a_2^3 - a_4\right)\right]x^4 + \ldots = 0 \end{aligned} \tag{10}$$

Each term's factor must be 0 for the left of Eq. (10) in order to keep their sum equal to 0, that is