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# On strength analysis of highly porous materials within the framework of the micropolar elasticity

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#### Abstract

We discuss the finite element approach to modelling of static deformations of porous materials such as foams, beam lattices, and others within the linear micropolar elasticity. It is known that the micropolar elasticity may be used for microstructured solids and fluids since it can forecast size-effect near geometrical singularities such as holes, notches, small contact areas of two solids. Within the micropolar elasticity the translational and rotational interactions of the material particles can be taken into account. Here we present the recent developments in the theory of finite elements calculations for micropolar solids in order to capture the stress behaviour in the vicinity of geometric singularities such as holes, notches, imperfections or contact areas. The fundamental equations of the micropolar isoparametric element and its implementation in ABAQUS are introduced. The solutions of few 3D benchmark problems of the micropolar elasticity are given. Among them are analysis of stresses and couple stresses near notches and holes, contact problem of parabolic stamp and half space. The main attention is paid to modelling of interaction between a biodegradable porous implant and a trabecular bone. Comparison of classical and micropolar solutions is discussed. Numerical tests have shown that couple stress appears almost in the vicinity of geometrical singularities than classical theory of elasticity are given.

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Keywords: micropolar elasticity; finite element method; foams; porous materials; bones

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#### 1. Introduction

Nowadays the interest grows to generalized models of continuum in order to model complex behavior of such microstructured materials as foams, bones and other porous and cellular materials. Among many generalized model the micropolar elasticity plays an important role, see Eringen (1999), Eremeyev et al. (2013). It can capture size-effect well-established for nanomaterials (Liebold and Müller (2015)), it also inherits rotational interactions and moments used in structural mechanics, see Goda and Ganghoffer (2015). The micropolar elasticity was proposed by Cosserat brothers more than hundred years ago and also found many applications for modeling of such materials as masonries, magnetic fluids, composites, etc., see Yang and Lakes (1982), Lakes (1986), Trovalusci et al. (2015), Eremeyev and Pietraszkiewicz (2012), Eremeyev et al. (2013). Within the micropolar elasticity two kinematically independent fields of translations and rotations and the stress and couple stress tensors are introduced.

Effective solution of boundary-value problems for micropolar solids requires development of advanced numerical code such as the finite element method and its implementation in efficient software. In particular, some commercial FEM software gives the possibility to use extended model of continuum applying so-called user defined elements and user defined material procedures. Here we discuss the implementation of new micropolar finite elements in ABAQUS.

#### 2. Governing Equations of the Linear Micropolar Elasticity

Following Eringen (1999), Eremeyev et al. (2013). we recall the basic equations of the linear micropolar elasticity. For simplicity we restrict ourselves by isotropic solids. The kinematic of a micropolar solid is described by two fields that are the field of translations ui and the field of rotations  $\theta$ , i=1,2,3. The latter is responsible for the description of moment (rotational) interactions of the material particles. Hereinafter the Latin indices take on values 1, 2, or 3 and we use the Einstein summation rule over repeating indices. The equilibrium equations take the form

$$t_{ji,j} + f_i = 0, \ m_{ji,j} + e_{imn}t_{mn} + c_j = 0, \tag{1}$$

where  $t_{ij}$  and  $m_{ij}$  are the Cartesian components of the nonsymmetric stress and couple stress tensors, respectively,  $e_{ijk}$  is the permutation symbol (Levi-Civita third-order tensor), and  $f_j$  and  $c_j$  are external forces and couples. Notation  $a_{,j}$  means the partial derivative of a with respect to Cartesian coordinate  $x_j$ .

The static and kinematic boundary conditions have the following form

$$n_{i}t_{ij}\Big|_{A_{i}} = \phi_{j}, \quad n_{i}m_{ij}\Big|_{A_{i}} = \eta_{j}, \quad u_{i}\Big|_{A_{u}} = u^{0}_{i}, \quad \theta_{i}\Big|_{A_{u}} = \theta^{0}_{i}, \quad (2)$$

where  $n_i$  is the components of the external normal to the boundary  $A = A_i U A_u$ ,  $\phi_j$  and  $\eta_j$  are external forces and couples prescibed on  $A_t$ , and  $u_i^{0}$  and  $\theta_i^{0}$  are given on  $A_u$  surface fields of translations and rotations, respectively.

Within the linear Cosserat continuum the constitutive relations for stresses and couple stresses can be represented as linear tensor-valued functions of strain  $\varepsilon_{ij}=u_{j,i}-e_{ijn}$   $\theta_n$ ,  $\kappa_{ij}=\theta_{j,i}$ . For micropolar elasticity we modified the Voigt notation as follows

$$\{\sigma_M\} = [\mathbf{C}]\!\{\varepsilon_M\}, \ [\mathbf{C}] = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix}, \ \{\sigma_M\} = \begin{bmatrix} \mathbf{T} \\ \mathbf{M} \end{bmatrix}, \ \{\varepsilon_M\} = \begin{bmatrix} \mathbf{E} \\ \mathbf{K} \end{bmatrix}, \tag{3}$$

where

$$\{\mathbf{T}\} = \{t_{11}, t_{22}, t_{33}, t_{12}, t_{21}, t_{23}, t_{32}, t_{13}, t_{31}\}^T,$$

$$(4)$$

$$\{\mathbf{M}\} = \{m_{11}, m_{22}, m_{33}, m_{12}, m_{21}, m_{23}, m_{32}, m_{13}, m_{31}\}^{T},$$
(5)

$$\{\mathbf{E}\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{23}, \varepsilon_{32}, \varepsilon_{13}, \varepsilon_{31}\},$$
(6)  
$$\{\mathbf{K}\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{23}, \varepsilon_{32}, \varepsilon_{13}, \varepsilon_{31}\},$$
(7)

$$(\mathbf{x}) = (\mathbf{x}_{11}, \mathbf{x}_{22}, \mathbf{x}_{33}, \mathbf{x}_{12}, \mathbf{x}_{23}, \mathbf{x}_{32}, \mathbf{x}_{13}, \mathbf{x}_{31}) , \qquad (7)$$

with  $18 \times 18$  stiffness matrix [C],  $9 \times 9$  matrices A and B which are not shown here. The exact form of [C] is given in Eremeyev and Pietraszkiewicz (2016), Eremeyev et al. (2013), Eremeyev et al. (2016a).

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