Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 3 (2017) 468–476

Structural Integrity Procedia

www.elsevier.com/locate/procedia

XXIV Italian Group of Fracture Conference, 1-3 March 2017, Urbino, Italy

On the fracture processes of cutting

P. Ståhle[a], A. Spagnoli[b,*], M. Terzano[b]

[a]*Division of Solid Mechanics, Lund University, SE-221 00 Lund, Sweden*
[b]*Department of Engineering and Architecture, University of Parma, Viale Usberti 181/A, 43124 Parma, Italy*

Abstract

The process of cutting is treated as a fracture mechanical process. For an elliptic rigid wedge pressed into an elastic material, fracture may occur as an autonomous process if the tip of the wedge is sufficiently blunt or is affected by the geometry of the wedge if the tip is sharp. The conditions leading to the former or the latter case is obtained as a relation between the wedge tip radius, the fracture toughness and the modulus of elasticity. These limits and the intermediate states are discussed. The implications of the drastic changes of the mechanical state of the near tip region when the wedge edge is sharp are also discussed.

Keywords: Cutting; fracture processes; linear elastic; fracture toughness.

1. Introduction

The mechanics of cutting has been attracting much attention in the literature, with the aim of addressing different technical problems (Williams and Patel, 2016; Williams et al., 2016) ranging from metal machining (Williams, 1998; Williams et al., 2010), to cutting of soft solids, such as biological tissues, foodstuffs or elastomeric materials (Goh et al., 2005; McCarthy et al., 2007, 2010). Cutting is also used as an experimental method for determining fracture toughness of polymers (Patel et al., 2009). Typically, cutting process is characterized by an indentation stage followed by a stage where the target material undergoes a progressive separation. The mechanical response in both stages is governed by the cutting tool shape (blade), the target material and the cutting rate. In the present paper, the quasi-static fracture stage is studied by means of a simple model which can be handled analytically, where the blade is modelled as a rigid wedge inserted into an elastic plate.

2. The wedge model for the cutting tool

Let us consider a large plate with a single centred cut of length $2a$. A cartesian coordinate system x, y is introduced in the centre of the plate so that the cut covers the region $|x| \leq a$, $y = 0$. An elliptic wedge with major and minor

* Corresponding author. Tel.: +39-0521-905927 ; fax: +39-0521-905924.
E-mail address: spagnoli@unipr.it

Copyright © 2017 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of the Scientific Committee of IGF Ex-Co.
10.1016/j.prostr.2017.04.063

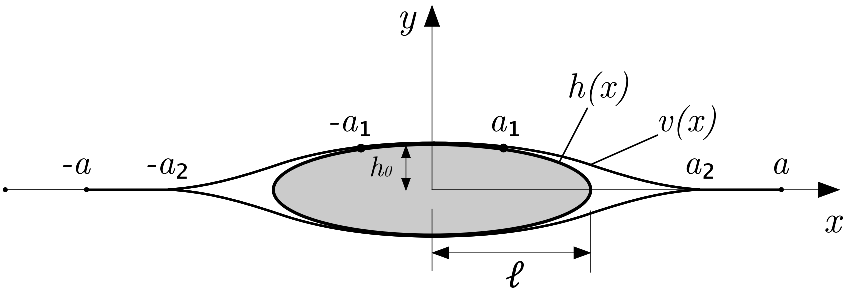

Fig. 1. An elliptic rigid wedge $|x| \leq \ell$ is inserted in a straight cut $|x| \leq a$, in a very large elastic plate. The wedge imposes a known displacement $h(x)$ in the region $|x| \leq a_1$ and traction free surfaces in the regions $a_1 < |x| < a_2 \leq a$, where contact between the cut surfaces is lost. Displacements vanish in $a_2 \leq |x| \leq a$.

semiaxes of length, respectively, ℓ and h_0, is inserted in the centre of the crack (Fig. 1). Its shape is expressed by

$$h(x) = h_o \sqrt{1 - (x/\ell)^2} \tag{1}$$

In order to define the wedge sharpness, we introduce the radius of curvature ρ at the wedge tip, which is equal to h_o^2/ℓ. A sharpness parameter θ can be defined if one divides ρ by a material-based length parameter. In absence of any inherent material length, the tip radius R_c of a crack experiencing a critical condition is considered, defined as

$$R_c = \left(\frac{\pi}{2}\right)\left(\frac{K_{\mathrm{I}c}}{E}\right)^2 \tag{2}$$

where $K_{\mathrm{I}c}$ is the critical stess intensity factor and E is Young's modulus. Therefore we obtain

$$\theta = \frac{h_0^2}{\ell R_c} \tag{3}$$

The wedge is assumed to be sharp for $\theta < 1$ and blunt for $\theta > 1$.

We assume the wedge to be shorter or have the same length of the cut, i.e., $\ell \leq a$ and partly in contact with its surfaces, which are supposed to be friction free. The elastic plate is assumed to be stress free far away from the wedge. As a result of the contact between the wedge and the cut surfaces, normal tractions can only be compressive or absent, i.e., $\sigma_y \leq 0$, along $|x| \leq a_1$, while shear tractions $\tau_{xy}(x)$ are null at all points along $y = 0$ because of frictionless surfaces. Moreover, the wedge generates imposed displacements $v(x) = h(x)$ in $|x| \leq \ell$.

In view of this, a boundary value problem can be formulated. Considering the upper half of the plate, such a problem is defined by the following equations

$$v(x) = h(x), \quad \text{with the condition} \quad \sigma_y(x) \leq 0 \quad \text{for} \quad |x| \leq a_1 \tag{4}$$