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Designing two-dimensional metamaterials of controlled static and dynamic properties

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ABSTRACT

In the current work, we elaborate two-dimensional metamaterials of controlled anisotropy. To that scope, we employ diamond and octagon-shaped planar lattices with and without inner links. Using a dedicated homogenization technique, we derive closed-form expressions for the lattice's effective mechanical properties. We analyse the effect of the lattice's configuration on the metamaterial's effective static properties, identifying configurations with mechanical attributes desirable for morphing, biomedical and mechanical engineering applications. We thereafter compute the lattice's wave propagation characteristics, deriving a link between the metamaterials' static and dynamic properties. In particular, we analyse the longitudinal and shear wave phase velocity dependence on the lattice's geometric configuration. Thereupon, we identify architectural arrangements for which the phase velocity vanishes in certain propagation directions, exhibiting wave propagation isolation characteristics. We demonstrate that the detected isolation features can systematically arise for lattice architectural designs that yield highly anisotropic static properties (thus high material moduli ratios) and anti-auxetic material behaviours (thus non-negative Poisson's ratio values).

1. Introduction

The emergence of additive manufacturing in combination with the advancement of engineering analysis tools has led to a new paradigm in the design of materials, in which the organization of matter plays a central role [1,2]. A new class of artificial materials arose that exhibit static and dynamic properties typically not encountered in natural materials and have been named as metamaterials [3]. In the present context, we use the term metamaterials to characterize artificial materials that obtain their mechanical characteristics out of their inner architecture (topology) - commonly engineered in periodically arranged unit cells - rather than out of their chemical composition [4,5].

Up to now a considerable amount of works has been dedicated to the conception and analysis of auxetic metamaterials, therefore materials with negative Poisson's ratios [6]. In particular, the mechanical properties of chiral lattices, hexa- and tetrachiral cellular solids have been analysed [7,8]. Auxetics have found a wide range of applications in different fields, as for example in the aerospace and automotive industry, primarily due to their superior shear strength and reduced overall structural weight [9].

The Poisson's ratio offers a fundamental metric to compare the material performance [10]. While for isotropic materials the Poisson's ratio value is limited in the range of $-1 \leqslant \nu \leqslant 0.5$, anisotropic materials can well exceed these limits. The necessity to induce a certain degree of anisotropy in order to achieve non-conventional mechanical behaviours has been recognized in a series of engineering fields, amongst others in morphing wing engineering applications [11]. Thereupon, successful morphing in aerospace or wind energy engineering has been directly associated with the development and usage of materials that exhibit a combination of low stiffness and high Poisson's ratio in the one material direction to "minimize actuation energy", combined with a high stiffness in its perpendicular material direction to adequately support aerodynamic loads [12]. Certain honeycomb and hybrid accordion cellular solids have been shown to satisfy the previously described stiffness characteristics [13,14]. Creating metamaterials of controlled anisotropy tuned to be ultra-soft or ultra-stiff and lightweight has become increasingly important not only in morphing, but also in biomechanical, civil and mechanical engineering applications [15–19]. Three dimensional unit cells have been designed so as to yield a priori specified material stiffness ratios, using evolutionary structural optimization methods [20]. Moreover, different cubic-shaped lattices and origami lattices have been employed to achieve optimal bulk and shear moduli and controlled Young's

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modulus values [21,22]. Apart from network materials, highly anisotropic mechanical properties have been obtained using reinforced composite structures with stiffness ratios between two normal material directions ranging from some hundreds up to some thousands. Such composites were expected to be used in the design of vibration damping or actuation devices of superior properties [23].

Certain applications necessitate anisotropic material designs not only due to stiffness requirements but also because of their Poisson's ratio behaviour. Recently, graphene materials with near-zero Poisson's ratio values have been fabricated, noting their need in nano-engineering applications [24]. Furthermore, specific biological structural members such as tendons and ligaments exhibit Poisson's ratio values well above the isotropic limits [25,26]. Therefore, the reconstruction of injured tissues such as ligaments and tendons requires the development of biocompatible biosubstitute materials that can inherently mimic the mechanical response of the native tissue [27].

In addition to their unusual static mechanical properties, artificial materials can demonstrate dynamic characteristics not encountered in common engineering materials. Their wave propagation characteristics can be obtained computing their dynamic attributes at the scale of their representative building block [28,29]. A primal design objective has been to obtain material architectures that favour the development of bandgap frequency regions. Triangular and hexagonal honeycomb lattices have been identified as typical lattice configurations that isolate wave propagation over certain frequency ranges [30]. Tetrachiral honeycombs have allowed for a veering of certain modes with direct application in both passive and active vibration control [31]. Recently, the ability to control elastic waves using elastomeric porous material structures that harness folding mechanisms has been highlighted [32]. Moreover, deformation induced buckling has been explored as a novel approach to tune wave propagation [5]. The wave propagation characteristics are highly sensitive to the lattice topology, with re-entrant and regular hexagonal lattices to exhibit utterly different dynamical properties [33]. Non-centrosymmetric triangular and square lattices with lumped masses have been associated with the appearance of low frequency bandgaps [34]. For chiral lattices, high chirality angle values have been shown to allow for bandgaps between the optical and acoustic branches [35]. What is more, anti-tetrachiral materials of increased auxeticity have the ability provide full bandgap regions over certain wave propagation directions [36]. Apart from the lattice topology, geometric and material non-linearities affect both the directionality and band gap range of the propagating waves [37,38]. Equivalently, material viscosity attenuates waves resulting in metamaterials with enhanced acoustic properties [39].

In the current work, we analyse polygonal lattices of controlled anisotropy. In particular, we consider diamond and octagon shaped lattices with and without inner links. In Section 2, we employ the discrete homogenization method to obtain closed-form expressions of their effective mechanical response. We complement the static analysis with the study of the dynamic wave propagation characteristics in Section 3. Thereupon, we assess the impact of increased anisotropy and more specifically of high material moduli ratios and non-auxetic Poisson's ratio values on the lattice's wave propagation features and conclude in Section 4.

2. Effective static mechanical properties

2.1. Constitutive law in 2D

We consider a general orthotropic two-dimensional elastic effective constitutive law, defined as follows [40]:

$$\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\epsilon} \Rightarrow \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{Bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{Bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} \tag{1}$$

where ϵ_x, ϵ_y stand for the normal stress components and ϵ_{yx} for the shear stress. For planar lattices, no out of plane strains appear, so that the following strain and stress components vanish:

$$\epsilon_z = \epsilon_{xz} = \epsilon_{yz} = 0 \quad \textit{and} \quad \sigma_z = \sigma_{xz} = \sigma_{yz} = 0 \tag{2}$$

Using Eqs. (1) and (2), we compute the compliance matrix $\mathbf{S}$ as the inverse of the stiffness matrix $\mathbf{S} = \mathbf{C}^{-1}$. The moduli and Poisson's ratio values of the arising continuum are defined as follows [41,42]:

$$E_1^* = \frac{1}{S_{11}}, \qquad E_2^* = \frac{1}{S_{22}}, \qquad \nu_{12}^* = -\frac{S_{21}}{S_{11}}, \qquad \nu_{21}^* = -\frac{S_{12}}{S_{22}} \tag{3}$$

Using the definitions for the effective mechanical moduli E_1^*, E_2^*, G and Poisson's ratio values ν_{12}^* and ν_{21}^* of Eq. (3), we rewrite the elasticity matrix $\mathbf{C}$ of Eq. (1) as follows:

$$\boldsymbol{C} = \begin{Bmatrix} \frac{E_1^*}{1-\nu_{12}^*\nu_{21}^*} & \frac{\nu_{21}^* E_1^*}{1-\nu_{12}^*\nu_{21}^*} & 0 \\ \frac{\nu_{12}^* E_2^*}{1-\nu_{12}^*\nu_{21}^*} & \frac{E_2^*}{1-\nu_{12}^*\nu_{21}^*} & 0 \\ 0 & 0 & G \end{Bmatrix} \tag{4}$$

2.2. Discrete asymptotic homogenization method summary

We use a discrete homogenization method, developed to provide the full compliance or stiffness matrix of lattice architecture which can handle complex beam lattices in a systematic, algorithmic way [43,44]. The method bases its development on the virtual power form of the equilibrium expressions for the beam (b) nodal (n^i) forces and moments developed within the two-dimensional (Z^2) lattice's unit cell, as follows:

$$\begin{aligned} &\sum_{n^i\epsilon Z^2}\sum_{b\epsilon B_R} \mathbf{T}^{\epsilon b}(\mathbf{v}^{\epsilon}(O(b)) - \mathbf{v}^{\epsilon}(E(b))) = 0 \\ &\sum_{n^i\epsilon Z^2}\sum_{b\epsilon B_R} (\mathbf{M}^{\epsilon O(b)}\mathbf{w}^{\epsilon}(O(b)) + \mathbf{M}^{\epsilon E(b)}\mathbf{w}^{\epsilon}(E(b))) = 0 \end{aligned} \tag{5}$$

In the lattice's force and moment equilibrium expressions of Eq. (5), $\mathbf{T}^{\epsilon b}$ stands for the sum of the normal and transverse forces and $\mathbf{M}^{\epsilon}$ for the moments developed in each beam element. The virtual velocity and rotation fields developed at the origin (O) and extremities (E) of each beam are denoted $\mathbf{v}^{\epsilon}$ and $\mathbf{w}^{\epsilon}$ accordingly, ϵ being the ratio of the elementary cell's beam length l to the macroscopic lattice's length L ($\epsilon = l/L$). Unique nodes n^i are defined and numbered within the unit cell, while the asymptotic developments of the nodal velocity and rotation fields are characterized by the structure's tessellation through the relative integers δ_i, taking values in the subset $\delta_i \in [-1, 0, 1]$ (see Fig. 1).

For a two-dimensional problem, the equilibrium expressions of Eq. (5) lead to a system of $3n$ in total equations, n being the number of unit cell nodes. The asymptotic form of the static and kinematic variables of Eq. (5) allows for the nodal displacements and rotations to be expressed with respect to the applied macroscopic gradients of the displacements in a strain driven scheme, which define the deformation tensor $\boldsymbol{\epsilon}$ applied to the unit cell. By computing the stress vector contributions $\mathbf{S}^i$ for each node and summing over the unit cell domain, the effective continuum description ($\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\epsilon}$) of the weak equilibrium form is retrieved:

$$\mathbf{S}^i = \sum_{b\epsilon B_R} \mathbf{T}^{\epsilon b}\delta^{ib} \rightarrow \int_{\Omega} \mathbf{S}^i \frac{\partial \mathbf{v}(\lambda^{\epsilon})}{\partial \lambda^i} d\lambda = 0 \Longleftrightarrow \int_{\Omega} \boldsymbol{\sigma}^i \frac{\partial \mathbf{v}}{\partial x} dx = 0, \quad \boldsymbol{\sigma} = \frac{1}{g}\mathbf{S}^i \otimes \frac{\partial \mathbf{R}}{\partial \lambda^i} \tag{6}$$