



# Concurrent topology optimization of composite macrostructure and microstructure constructed by constituent phases of distinct Poisson's ratios for maximum frequency



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## ABSTRACT

This paper introduces a two-scale concurrent topology optimization method for maximizing the frequency of composite macrostructure that are composed of periodic composite units (PCUs) consisting of two isotropic materials with distinct Poisson's ratios. Interpolation of Poisson's ratios of different constituent phases is used in PCU to exploit the Poisson effect. The effective properties of the composite are computed by numerical homogenization and integrated into the frequency analysis. The sensitivities of the eigenvalue of macro- and micro-scale density are derived. The design variables on both the macro- and micro-scales are efficiently updated by the well-established optimality criteria methods. Several 2D and 3D illustrative examples are presented to demonstrate the capability and effectiveness of the proposed approach. The effect of the micro-scale volume fraction and Poisson's ratio of the constituent phases on the optimal topology are investigated. It is observed that higher frequency can be achieved at specific range of micro-scale level volume fraction for optimal composites than that obtained from structures made of individual base materials.

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## 1. Introduction

The optimization of frequency is of major importance for many engineering applications in aeronautics, astronautics and automotive industries. Amongst various methods, topology optimization techniques, as one of the most promising tools, have been developed in the past few decades to find the optimal layouts. A comprehensive review on topology methods for solving vibration problems in structural designs was presented in literature [1] including homogenization method [2–4], evolutionary structural optimization method [5,6], solid isotropic material with penalization method (SIMP) [7,8] and level set method [9]. Meanwhile, topology optimization combined with inverse homogenization technique also has broad applications in microstructural design of cellular and composite materials. More recently, to reduce the vibration level, Andreassen et al. [10] proposed a method to optimize the microstructure of viscoelastic composites, which maximized the damping capabilities of composites considering the manufacturability. Huang et al. [11] extended the bi-directional evolutionary structural optimization (BESO) method for designing

microstructures of viscoelastic composites with high damping characteristics. Jensen and Sigmund [12] employed the topology optimization method to design a T-junction in a photonic crystal waveguide based on band gap structure. Meng et al. [13] proposed a BESO design method for the photonic crystals with negative refraction properties.

However, the aforementioned research are restricted to individual macro-structural optimization or micro-scale material optimization, i.e. designing the macrostructures composed of homogeneous materials, or designing the microstructures for the expected or extremal properties individually. Rodrigues et al. [14] initially generalized a hierarchical optimization formulation for optimizing material distribution in both macrostructure and microstructure. Coelho et al. [15] extended this hierarchical computational procedure to 3D elastic structures. However, microstructural topologies may vary arbitrarily in space and result in high computational cost and less manufacturability. Liu et al. [16] proposed a concurrent topology optimization model where microstructure is assumed to be identical to each other in the macrostructure and periodically distributed. Independent relative densities for the macrostructure and microstructure are defined and coupled into an integral system through homogenization theory. The extension of this approach seeking for maximum primary

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frequency was given by Niu et al. [17]. Yan et al. [18,19] presented a concurrent topology optimization strategy for minimum compliance of thermo-elastic structures. In their numerical results, the porous material showed advantages in reducing the system compliance when simultaneously considering the mechanical and thermal loads. Guo et al. [20] presented a robust concurrent optimization formulation to find optimal material and structure simultaneously considering the uncertainties of loads. It was observed that the microstructures are generally isotropic and in a form of Kagome structure under such uncertainties. Huang et al. [21] utilized the BESO method for realizing the concurrent optimization design. Unambiguous configurations on both macro- and micro- scales are obtained within BESO framework. Zuo et al. [22] introduced a hierarchical design model containing multiple material phases on both the macrostructure and microstructure. Yan et al. [23,24] presented a concurrent optimization algorithm where only one total mass constraint on material usage was applied. Liu et al. [25] also proposed a concurrent topology optimization model to maximize natural frequency with a given mass. Xu et al. [26–28] extended the BESO method to concurrent topology optimization in regard to material distribution in macrostructure and periodic microstructure under harmonic, transient and random excitations. They also discussed the concurrent design of thermo-elastic structures composed of periodic multiphase materials [29]. Vicente et al. [30] presented concurrent topology optimization models for minimizing the frequency responses. Zhang and Sun [31] revealed the size effect of materials and structures in the integrated two-scale optimization approach. More recently, Xia et al. [32–34] proposed a FE<sup>2</sup> resolution framework where the nonlinearity was addressed for the concurrent design of materials and structures. Jia et al. [35] presented a hierarchical design of structures and multiphase material cells. In the model, the macro- and micro-scales design variables were linked by elemental phase density. The hierarchical optimization model of structures and multiphase cells is built under prescribed volume fraction and mass constraints.

Composite materials usually combine two or more constituent phases with significantly different characteristics, which may have preferable properties over conventional materials. Recent studies showed that the Poisson's ratio of base material played critical role on mechanical properties of composites. Liu et al. [36] observed significantly increased stiffness in two principle directions of staggered biocomposites when the Poisson's ratio of soft constituent phase approaching the incompressibility limit of 0.5. Similar increase in stiffness was found in laminate composites with alternating layers of materials with negative Poisson's ratio (NPR) and positive Poisson's ratio (PPR) [37–39]. Long et al. [40] proposed a topology optimization algorithm to acquire the maximum effective Young's modulus. Other similar research also proposed the methodology for designing high-stiffness composites by considering Poisson effect [41].

The above studies [36–41] focused on the design of high-stiffness composite constructed by constituent phases of distinct Poisson's ratios. However, topology optimization of macrostructure composed of such composites might not bring optimal solutions when considering complex boundary conditions. In this study, we employ topology optimization technique in the concurrent design of macrostructure and periodic microstructure of composites containing multiple phases with different Poisson's ratios. The rest of this paper is structured as follows. Section 2 formulates the concurrent topology optimization algorithm for maximizing the natural frequency of macrostructure. Section 3 describes the homogenization method for effective material properties and sensitivity analysis with respect to macro- and micro-scales design variables. Section 4 describes the filtering schemes to eliminate the numerical instabilities and the optimality criteria method. Sec-

tion 5 presents four numerical examples to validate the effectiveness of the proposed optimization method. Section 6 summarizes the main findings.

## 2. Concurrent topology optimization for maximum natural frequency

In this paper, it is assumed that the macrostructure of a composite which is composed of periodic composite units (PCUs) as indicated in Fig. 1. Both macrostructure and microstructure are discretized by finite element (FE). Each element on macro-scale or micro-scale level is assigned an exclusive relative density, i.e. macro-elemental density  $P_i$  ( $i = 1, 2, \dots, M$ ) or micro-elemental density  $r_j$  ( $j = 1, 2, \dots, N$ ), whose value is either 0 or 1, where  $M$  and  $N$  are the total number of elements in macrostructure and microstructure, respectively. In the PCU, when the  $j$ th element is occupied by phase 2,  $r_j = 0$ , while  $r_j = 1$  when it is occupied by phase 1.

The concurrent topology optimization aims at finding the maximum  $k$ th order frequency of the macrostructure. The optimization problem can be mathematically expressed as:

$$\text{Find: } \mathbf{X} = \{P_i, r_j\}, (i = 1, 2, \dots, M; j = 1, 2, \dots, N)$$

$$\text{Maximize: } \lambda_k$$

$$\text{Constraint I: } \mathbf{K}\mathbf{u}_k = \lambda_k \mathbf{M}\mathbf{u}_k$$

$$\text{Constraint II: } \sum_{i=1}^M P_i V_i / V^{mac} \leq f^{mac} \quad (1)$$

$$\text{Constraint III: } \sum_{j=1}^N r_j V_j / V^{mic} \leq f^{mic}$$

$$\text{Constraint IV: } P_{\min} \leq P_i \leq 1, r_{\min} \leq r_j \leq 1$$

where  $\lambda_k$  and  $\mathbf{u}_k$  denotes the  $k$ th order macrostructural eigenvalue and its corresponding eigenvector.  $\mathbf{K}$  and  $\mathbf{M}$  are the global stiffness and mass matrices of the macrostructure. Constraints II and III describe the volume fraction in macrostructure and microstructure respectively.  $V_i$  and  $V_j$  denote the volume of the  $i$ th macro-scale element or the  $j$ th micro-scale element, respectively.  $V^{mac}$  and  $V^{mic}$  denote the total volume of the macrostructure or the microstructure, respectively.  $f^{mac}$  and  $f^{mic}$  denote the prescribed volume fraction on macro- and micro-scale level, respectively. Constraint IV defines the bounds for design variable to ensure the non-singularity in numerical analysis. The minimum density  $P_{\min} = r_{\min} = 10^{-3}$  is chosen in this paper.

The global stiffness and mass matrices are assembled by the elemental stiffness matrix  $\mathbf{K}_i$  and mass matrix  $\mathbf{M}_i$ , respectively,

$$\mathbf{K} = \sum_{i=1}^M \mathbf{K}_i = \sum_{i=1}^M \int_{V_i} \mathbf{B}^T \mathbf{D}_i^{MA} \mathbf{B} dV_i = \sum_{i=1}^M \int_{V_i} \mathbf{B}^T (P_i^p \mathbf{D}^H) \mathbf{B} dV_i \quad (2)$$

$$\mathbf{M} = \sum_{i=1}^M \mathbf{M}_i = \sum_{i=1}^M \int_{V_i} \mathbf{N}^T \rho_i^{MA} \mathbf{N} dV_i = \sum_{i=1}^M \int_{V_i} \mathbf{N}^T (P_i \rho^H) \mathbf{N} dV_i \quad (3)$$

where  $\mathbf{B}$  and  $\mathbf{N}$  denote the strain-displacement matrices and shape function matrices on the macro-scale level.  $\mathbf{D}_i^{MA}$  and  $\rho_i^{MA}$  denote the elasticity matrix and density for the  $i$ th element in macrostructure.  $\mathbf{D}^H$  and  $\rho^H$  denote the effective elasticity matrix and homogenized density computed through the standard homogenization theory within PCU.  $p$  is penalization power for elemental stiffness matrix. In this study, we found that  $p = 3$  could have good convergence and clear topologies. The localized mode or pseudo mode is the eigenmode occurred in domains occupied by the elements with low densities. Usually the corresponding frequencies are very low, which are undesirable in the frequency optimization problem [7]. In local mode, the elemental stiffness can be modified to keep the

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