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The effects of different quantum feedback types on the tightness of the variance-based uncertainty

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ABSTRACT

The effect of the quantum feedback on the tightness of the variance-based uncertainty, the possibility of using quantum feedback to prepare the state with a better tightness, and the relationship between the tightness of the uncertainty and the mixedness of the system are studied. It is found that the tightness of Schrodinger-Robertson uncertainty (SUR) relation has a strict liner relationship with the mixedness of the system. As for the Robertson uncertainty relation (RUR), we find that the tightness can be enhanced by tuning the feedback at the beginning of the evolution. In addition, we deduce that the tightness of RUR has an inverse relationship with the mixedness and the relationship turns into a strict linear one when the system reach the steady state.

1. Introduction

The uncertainty principle which is similar to the quantum entanglement is one of the most remarkable characters of quantum mechanics as well as a fundamental departure from the principle of classical physics [1–7]. Any pair of incompatible observables complies with a certain form of uncertainty relationship, the constraint of which sets the ultimate bound on the measurement precision achievable for these quantities. The conventional variance-based uncertainty relations which is first deduced by Heisenberg in the circumstance of position and momentum possess a clear physical conception and still find a variety of applications in quantum information science, such as entanglement detection [4,8], quantum spin squeezing [9–14], and even quantum metrology [15–17]. Therefore the improvement of the tightness and the lower bound of the uncertainty becomes very important [18]. Robertson uncertainty relation (RUR) is the most famous form among them, which reads [2]:

$$(\Delta A)^2(\Delta B)^2 \geq \left|\frac{1}{2i}\langle[A,B]\rangle\right|^2, \tag{1}$$

where the standard deviation $\Delta O$and expectation value $\langle O\rangle$ are taken over the state ρ with $O\in\{A,B\}$. Meanwhile, it is worth noting that the RUR can be derived from another strengthened inequality, the Schrodinger-Robertson uncertainty relation (SUR) [19,20]:

$$(\Delta A)^2(\Delta B)^2 \geq \left|\frac{1}{2i}\langle[A,B]\rangle\right|^2 + \left|\frac{1}{2}\langle\{\check{A},\check{B}\}\rangle\right|^2. \tag{2}$$

Here I is the identical operator and $\check{O} = O - \langle O\rangle I$. The uncertainty inequalities of the types of Eq. (1) and Eq. (2) are often known as Heisenberg-type and Schrodinger-type uncertainty relations, respectively.

Quantum feedback [21], as a means of controlling the system, is more and more valued by researchers in the preparation of the special state. Thus it would be of great interest to investigate the effect of the quantum feedback control on the tightness of the uncertainty. In this paper we mainly study the influence of the feedback on the properties of RUR and SUR, and the relationship between the tightness of the uncertainty and the mixedness of the system. An outline of the paper is as follows. In Sec. 2, the physical model a qubit interacting with a dissipative cavity which provide the feedback to the qubit is introduced. The effects of the feedback on the tightness and mixedness are studied in Sec. 3, Finally, Sec. 4 is devoted to the discussion and conclusion.

2. The physical model

The physical model of a single atom resonantly coupled to a single-mode cavity, which is damped with decay rate κ, will be introduced. As

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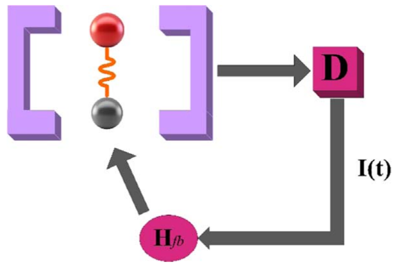

Fig. 1. Schematic view of the model, the feedback Hamiltonian is applied to the atoms according to the homodyne current I(t) derived from detector D.

shown in Fig. 1.

In the model, we will consider the Markovian feedback [21], with the control Hamiltonian $H_{fb} = I(t)F$, where $I(t)$ is the signal from the homodyne detection of the cavity output. In the homodyne-based scheme, the detector registers a continuous photocurrent and the feedback Hamiltonian is constantly applied to the system. The master equation is read as [22,23]

$$\frac{d\rho}{dt} = -i\left[\frac{1}{2}(\sigma^{+}F+F\sigma_{-}),\, \rho\right] + D(\sigma_{-}-iF)\rho, \tag{3}$$

where ρ is the density matrix of the qubit, and $D(\mathcal{A})\rho = \mathcal{A}\rho\mathcal{A}^{+} - (\mathcal{A}^{+}\mathcal{A}\rho + \rho\mathcal{A}^{+}\mathcal{A})/2$ represents the irreversible evolution induced by the interaction between the system and the environment.

3. The effect of feedback on uncertainty

An arbitrary Hermitian operator of qubit systems can be denoted by $\varepsilon_1 I+\varepsilon_2\vec{a}\vec{\sigma}$, where $\{\varepsilon_1, \varepsilon_2\}$ are real parameters, $\vec{a}\in R_3$ are unit vectors, and $\vec{\sigma} = (\sigma_x\sigma_y\sigma_z)$ are standard Pauli matrices. If we choose the identity operator I as feedback, the system will not be affected by tuning the feedback due to the properties of identity operator. Therefore, in order not to lose generality, two different Hermitian operators $\lambda(\sin(\beta)\sigma_x+\cos(\beta)\sigma_y)$ and $\mu\sigma_z (\lambda \in (-1, 1), \mu\in(-1, 1)$, and $\beta\in(0, 2\pi))$are selected as the feedback operators to study the effect of different feedback types on the uncertainty. In order to research the tightness of the RUR and SUR, we define:

$$U = \Delta\hat{A}^2\Delta\hat{B}^2 - \frac{1}{4}|[\hat{A}, \hat{B}]|^2 \tag{4}$$

$$W = \Delta\hat{A}^2\Delta\hat{B}^2 - \frac{1}{4}|[\hat{A}, \hat{B}]|^2 - \left|\frac{1}{2}\left\{\check{A}, \check{B}\right\}\right|^2 \tag{5}$$

It is easy to see that the smaller the value of $U(W)$ is, the better the tightness of RUR (SUR) is. For the density matrix ρ, the state is a pure one when $\mathrm{Tr}(\rho^2) = 1$, and $\mathrm{Tr}(\rho^2)<1$ for the mixed one. Denoting $1 - \mathrm{Tr}(\rho^2)$ by Υ, therefore the value of Υ can be employed to detect the mixedness of qubits states. We can deduce that the bigger the value of Υ is the bigger the mixedness of ρ is. In the following, we mainly focus on the effect of feedback and mixedness on the uncertainty relation from the aspect of tightness. We take $\hat{A} = \sigma_x$ $\hat{B} = \sigma_z$ and choose superposition state $|\varphi_0\rangle = \cos(\alpha)|g\rangle + \sin(\alpha)|e\rangle$ as the initial state in the following.

3.1. The feedback $\lambda(\sin(\beta)\sigma_x + \cos(\beta)\sigma_y)$

The feedback $\lambda(\sin(\beta)\sigma_x + \cos(\beta)\sigma_y)$ will be studied in this subsection. Substituting $F = \lambda(\sin(\beta)\sigma_x + \cos(\beta)\sigma_y)$ into Eq. (3), one can obtain the density matrix:

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & \rho_{12}(t) \\ \rho_{12}{}^{*}(t) & 1 - \rho_{11}(t) \end{pmatrix} \tag{6}$$

with the elements

$$\rho_{11}(t) = \frac{(1 + 2e^{\mathcal{T}t}\lambda^2-(1 + 2\lambda^2)\cos(2\alpha) + 4\lambda\cos(\beta)\sin(\alpha)^2)}{2\mathcal{T}e^{\mathcal{T}t}} \tag{7}$$

$$\rho_{12}(t) = -\frac{\sin(2\alpha)(-2e^{i\beta}\xi + 2ie^{2t\lambda\mathcal{P}}(1+e^{i\beta}\lambda)\sin(\beta))}{4\sqrt{O}\mathcal{P}} \tag{8}$$

in which $O = \exp(t+4t\lambda^2+4t\lambda\cos(\beta))$, $\mathcal{P} = \lambda+\cos(\beta)$, $\mathcal{T} = 1 + 2\lambda^2+2\lambda\cos(\beta)$,$\xi= 1 + \lambda\cos(\beta)$ After a simple calculation, we have

$$(\triangle\hat{A})^2 = 1 - \frac{\sin(2\alpha)^2(2\cos(\beta)+\lambda(1 + \cos(2\beta) + 2e^{2t\lambda\mathcal{P}}\sin(\beta)^2))^2}{4O\mathcal{P}^2} \tag{9}$$

$$(\triangle\hat{B})^2 = 1 - \frac{[\mathcal{T}\cos(2\alpha) + (e^{\mathcal{T}t}-1)(1 + 2\lambda\cos(\beta))]^2}{\mathcal{T}^2e^{2\mathcal{T}t}} \tag{10}$$

$$|\langle[\hat{A}, \hat{B}]\rangle|^2 = \frac{4(1 + \lambda\cos(\beta))^2\sin(2\alpha)^2\sin(\beta)^2(e^{2\mathcal{P}t\lambda}-1)^2}{O\mathcal{P}^2} \tag{11}$$

$$|\langle\{\check{A}, \check{B}\}\rangle|^2 = \frac{4((e^{\mathcal{T}t}-1)(1 + 2\lambda\cos(\beta)) + \cos(2\alpha)\mathcal{T})^2\sin(2\alpha)^2\mathcal{G}^2}{\mathcal{P}^2\mathcal{T}^2e^{3\mathcal{T}t}} \tag{12}$$

$$\Upsilon = \frac{(-\mathcal{P}^2\ell(\ell-2\mathcal{T}e^{\mathcal{T}t})-\mathcal{T}^2e^{t}\sin(2\alpha)^2(\xi^2 + e^{2\mathcal{P}t\lambda}(-2\xi + e^{2\mathcal{P}t\lambda}(\mathcal{T} - \lambda^2))\sin(\beta)^2))}{2e^{2\mathcal{T}t}\mathcal{T}^2\mathcal{P}^2} \tag{13}$$

with $\ell = 1 + 2\lambda^2\exp(\mathcal{T}t) - (1 + 2\lambda^2)\cos(2\alpha) + 4\lambda\cos(\beta)\sin(\alpha)^2$ and $\mathcal{G} = \mathcal{P}\cosh(t\lambda\mathcal{P}) - (\cos(\beta) + \lambda\cos(2\beta))\sinh(t\lambda\mathcal{P})$. According to the above formulas, one can deduce that:

$$W = \Delta\hat{A}^2\Delta\hat{B}^2 - \frac{1}{4}|\langle[\hat{A}, \hat{B}]\rangle|^2-\left|\frac{1}{2}\langle\{\check{A}, \check{B}\}\rangle\right|^2 = 2\Upsilon \tag{14}$$

which means the tightness of the SUR has a strict linear relationship with the mixedness of the system and the less the mixedness is the better the tightness of the SUR is. Therefore we can investigate the tightness of the SUR by the evolution of the mixedness.

In the following, the influence of the feedback on the tightness of the RUR and the mixedness of the system will be investigated and we firstly concentrate on the feedback $\lambda\sigma_x$, namely take $\beta = \pi/2$. Making use of Eqs. (9), (10), (11) and (13), one can obtain the evolution of U and Υ with respect to time for different initial state in Fig. 2, here we take $\lambda = 1$.

It is easy to see from Fig. 2 that choosing the superposition state $(|g\rangle+|e\rangle)/\sqrt{2}$ can be more effective to enhance the tightness of uncertainty and reduce the mixedness than choosing excited and ground one as initial state. In addition, it's easy to find that evolution of mixedness has almost the same structure with the one of U, which means there is a proportional relationship between the value of U and Υ. In other words, the tightness of RUR is inversely proportional to the mixedness.

As we can see from Fig. 2 the tightness and the mixedness of the steady state has nothing to do with the initial state we choose. Let $t \to \infty$ and make use of Eqs. (4), (9), (10), (11) and (13) one can acquire $U_{t\to\infty} = 2\Upsilon_{t\to\infty}$, that is to say, the tightness of RUR has a strict linear relationship with the mixedness at the case that the system is in steady state. The expressions and evolution of $U_{t\to\infty}$ and $\Upsilon_{t\to\infty}$ are given as

$$U_{t\to\infty} = \lim_{t\to\infty}(\triangle\hat{A}^2\triangle\hat{B}^2-\frac{1}{4}|\langle[\hat{A}, \hat{B}]\rangle|^2) = 1 - \frac{1}{(1 + 2\lambda^2)^2} \tag{15}$$

$$\Upsilon_{t\to\infty} = \lim_{t\to\infty}\Upsilon = \frac{1}{2}(1 - \frac{1}{(1 + 2\lambda^2)^2}) \tag{16}$$

As shown in Fig. 3, the tightness and mixedness are only affected by the value of λ, which represents the feedback strength. In addition, we can see that the tightness reaches the optimum value while the mixedness reaches the minimum value when the system has no

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