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Integer quantum Hall effect in a triangular-lattice: Disorder effect and scaling behavior of the insulator-plateau transition

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ABSTRACT

We investigate numerically the integer quantum Hall effect in a three-band triangular-lattice model. The three bands own the Chern number C=2,−1,−1, respectively. The lowest topological flat band carrying Chern number C=2, which leads to the Hall plateau $\sigma_H = 2(e^2/h)$. This Hall plateau is sensitive to the disorder scattering and is rapidly destroyed by the weak disorder. Further increasing the strength of disorder, the gap of density of states always disappears before the vanishing of the corresponding Hall plateau. The scaling behavior of quantum phase transition between an insulator and a quantum Hall plateau is studied. We find that the insulator-plateau transition becomes sharper with increasing the size of system. Due to the different of edge states, the critical energy E_{c1} gradually shifts to the center of Hall plateau while E_{c2} is unaffected with increasing the disorder strength.

1. Introduction

The integer quantum Hall effect (IQHE) is discovered experimentally by Kliziting in 1980 [1], in which the transverse Hall conductivity of a two-dimensional (2D) system of electrons is found to have plateaus in the presence of a strong perpendicular magnetic field [1,2]. Immediately after this discovery, scientists from various disciplines were launched into a frenzy of activity to understand the underling physics and also to explore its technological importance in designing different electronic devices. The first theoretical work on this effect was put forward by Laughlin [3] based on a gauge-invariance argument, and further theoretical elaboration was made possible through the picture of edge states by Halperin [4]. However, it is also natural to explain the realization of the IQHE by explicit calculations based upon the linear-response Kubo formula for periodic systems [5]. The electron energy spectrum is split into discrete Landau levels. If the Fermi level lies exactly between two Landau levels, the Hall conductance can be labeled by topological Chern number in units of e^2/h and be accurately quantized to an integer.

The Anderson localization theory [6,7] predicts that non-interacting electrons in 2D disordered system in the absence of a magnetic field are generally localized with time-reversal and spin rotational symmetries. However, the time-reversal symmetry is broken by external magnetic field, and a series of Landau bands appear due to disorder. The interplay between magnetic field and disorder is an essential issue for the phenomenon of IQHE. There are many theoretical studies focusing on the existence of a plateau-insulator transition [8–11] in 2D electronic system with an external magnetic field. The energy spectrum in a uniform magnetic field for a square lattice was investigated by Hofstadter [12] and since then the electronic properties of 2D periodic lattice structures immersed in a uniform magnetic field have drawn a lot of interests and been hot topics among theoreticians [11,13–22]. Sheng et al. [11,14–16] studied the IQHE at strong disorder and weak magnetic field in the tight-binding lattice model, where the systematic float-up and merging picture for extended level was found. The Scaling behavior of the quantum phase transition between an insulator and a quantized Hall plateau state or between two adjacent plateau states has been examined by realizing landau levels. A nonzero integer Chern number distinguishes an extended state from a localized state (Chern number is zero) in the system. So they suggested that topological characterization in terms of Chern integers provide a simple physical explanation and gives a qualitative difference between the lattice and continuum models.

Though most of the theoretical studies based on the lattice structure with a uniform magnetic field has been performed, a very few theoretical works [23–25] are available where the multi-band lattice model has been used. However, the effects of the disorder strength on the transition between an insulator and a quantum Hall plateau and the scaling behavior of the quantum phase transition have not been studied in the multi-band lattice model. We wish to address these

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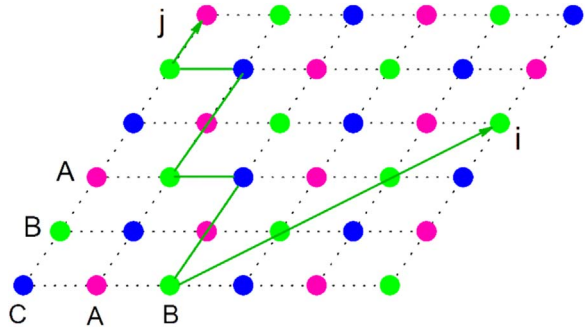

Fig. 1. Schematic diagram of a 2D triangular-lattice with three single-particle bands labeled by pink (A), green (B) and blue (C)circle dots, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

issues in the present work.

In this paper, we will study in detail numerically the Hall conductance and density of states (DOS) using the exact diagonalization method and Kubo formula. The model and theoretical method are described in the next section while the numerical results and related discussions are given in Section 3. In Section 4 our results are briefly summarized.

2. The model and theoretical method

We consider a 2D triangular-lattice as shown in Fig. 1. It is characterized by three single-particle bands (A, B and C) per unit cell. Creation (a_{ij}^+, b_{ij}^+, c_{ij}^+) and annihilation (a_{ij}, b_{ij}, c_{ij}) operators are introduced, where i(j) stands the site index along the direction of arrow. The tight-binding (TB) Hamiltonian is employed to describe our system and it reads as

$$\begin{aligned} H = \sum_{i,j} & w_{ij}(a_{ij}^+ a_{ij} + b_{ij}^+ b_{ij} + c_{ij}^+ c_{ij}) + [t(b_{i,j+2}^+ + b_{i,j-1}^+ + b_{i+1,j-1}^+ - c_{i,j+1}^+ \\ & - e^{2i\phi}c_{i+1,j+1}^+ - e^{-2i\phi}c_{i+1,j-2}^+) + t'e^{i\phi}(a_{i+1,j}^+ + a_{i-1,j+3}^+ + a_{i,j-3}^+) \\ & + t'e^{-i\phi}(a_{i-1,j}^+ + a_{i+1,j-3}^+ + a_{i,j+3}^+)]a_{ij} + [t(a_{i,j+1}^+ + a_{i-1,j+1}^+ + a_{i,j-2}^+ \\ & + c_{i+1,j-1}^+ + e^{-2i\phi}c_{i,j+2}^+ + e^{2i\phi}c_{i,j-1}^+) + t'e^{i\phi}(b_{i-1,j}^+ + b_{i,j+3}^+ + b_{i+1,j-3}^+) \\ & + t'e^{-i\phi}(b_{i+1,j}^+ + b_{i,j-3}^+ + b_{i-1,j+3}^+)]b_{ij} + [t(-a_{i,j-1}^+ - e^{2i\phi}a_{i-1,j+2}^+ \\ & - e^{-2i\phi}a_{i-1,j-1}^+ + b_{i-1,j+1}^+ + e^{2i\phi}b_{i,j-2}^+ + e^{-2i\phi}b_{i,j+1}^+) \\ & - t'(c_{i+1,j}^+ + c_{i-1,j}^+ + c_{i+1,j-3}^+ + c_{i,j+3}^+ + c_{i,j-3}^+ + c_{i-1,j+3}^+)]c_{ij}, \end{aligned} \tag{1}$$

where the nearest-neighbor and next-nearest-neighbor hoppings are considered in Eq. (1), $w_{ij}$is a disorder potential energy uniformly distributed between $(-W/2, W/2)$ on site (note that the disorder strength W here is as big as that defined in Ref. [11]), t (t') is the hopping integral between the nearest (next-nearest) neighboring sites, ϕ is the azimuthal angle for the vector connecting hopping.

Assuming that the lattice is composed of N_1 and N_2 number of sites along the i- and j-directions, respectively, where N_2 is a multiple of 3, and the total number of lattice sites is $N = N_1 \times N_2$. With the choice of $W = 0$ and employing the Fourier transform $c(k_x, k_y) = \frac{1}{\sqrt{N_1 N_2}}\sum_{i_x j_y} c_{ij} e^{i(k_x i_x + k_y j_y)}$, the k-representation of Hamiltonian is described by a 3×3 matrix as following:

$$H = \sum_{\vec{k}} (a_{\vec{k}}^+ \quad b_{\vec{k}}^+ \quad c_{\vec{k}}^+) \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{31} \end{pmatrix} \begin{pmatrix} a_{\vec{k}} \\ b_{\vec{k}} \\ c_{\vec{k}} \end{pmatrix}. \tag{2}$$

Here,

$$\begin{cases} H_{11} = t'[2\cos(k_x + \phi) + 2\cos(k_x - 3k_y - \phi) + 2\cos(3k_y - \phi)] \\ H_{22} = t'[2\cos(k_x - \phi) + 2\cos(k_x - 3k_y + \phi) + 2\cos(3k_y + \phi)] \\ H_{33} = t'[2\cos k_x + 2\cos 3k_y + 2\cos(k_x - 3k_y)] \\ H_{12} = H_{21}^* = t(e^{ik_y} + e^{i(-k_x + k_y)} + e^{-2ik_y}) \\ H_{13} = H_{31}^* = -t(e^{i(-k_x + 2k_y + 2\phi)} + e^{-i(k_x + k_y + 2\phi)} + e^{-ik_y}) \\ H_{23} = H_{32}^* = t(e^{i(k_y - 2\phi)} + e^{i(-k_x + k_y)} + e^{2i(-k_y + \phi)}) \end{cases}.$$

we adopt the parameters $t = 1$, $t' = 1/4$, $\phi = 2\pi/6$, and use the periodic boundary conditions. The energy spectrum of the triangular-lattice is shown in Fig. 2(a) and (b). It is found that the topological flat band states are well created and the two gaps are verified between sub-bands.

After the numerical diagonalization of the Hamiltonian (Eq. (1)), the Hall conductance can be calculated by using the Kubo formula [5,13]:

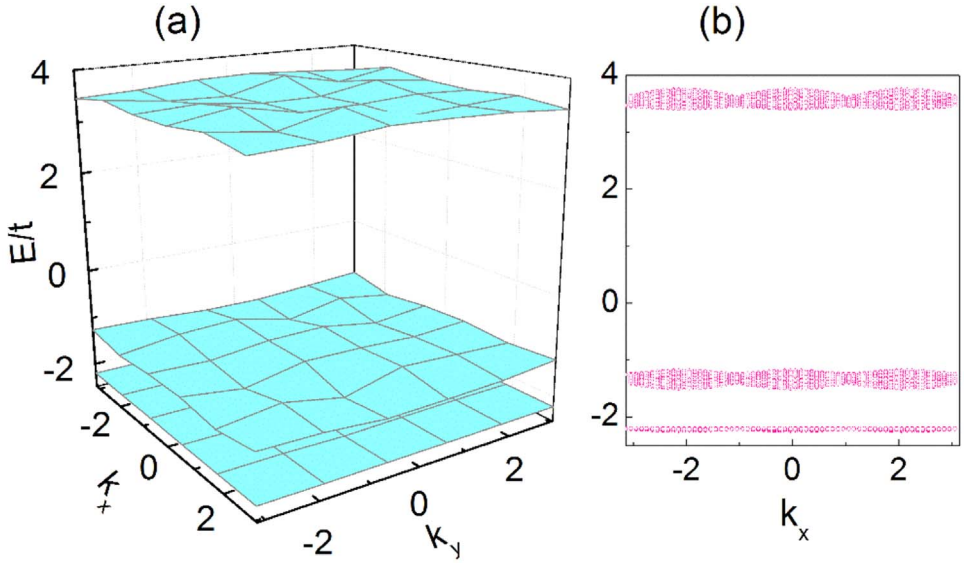

Fig. 2. The energy spectrum of the 2D triangular-lattice model. (b) is the cut plane of (a). The lowest topological flat band is obtained.