Disordered stabilization of stochastic delay systems: The disorder-dependent approach

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Abstract: In this paper, a general stabilization problem of stochastic delay systems is realized by a disordered controller and studied by exploiting the disorder-dependent approach. Different from the traditional results, the stabilizing controller here experiences a disorder between control gains and system states. Firstly, the above disorder is described by the robust method, whose probability distribution is embodied by a Markov process with two modes. Then, by exploiting a disorder-dependent Lyapunouv functional, two respective conditions for the existence of such a disordered controller are provided with LMIs. Finally, a numerical example is exploited to demonstrate the effectiveness and superiority of the proposed methods.

Key Words: stochastic delay systems; disorder; disorder-dependent approach; Markov process; robust; stabilization

1 INTRODUCTION

It is very known that time delay is commonly encountered in variously practical dynamical systems, such as chemical systems, heating systems, biological systems, networked control systems (NCSs), telecommunication and economic systems, and so on. Due to the presence of time delay in such practical systems, many negative effects, for example, oscillation, instability and poor performance, could be caused. Motivated by these facts, various research topics of time-delayed systems have been considered. By investigating the existing results, it is found that they are mainly classified into two classes: delay-dependent and -independent ones.During the past decades, many important results on all kinds of delay systems have emerged, such as stability analysis [1, 2], stabilization [3, 4, 5],dissipative and passive control [6, 7, 8], output control [9], H_∞ control and filtering [10, 11, 12, 13], state estimation [15, 16, 17], synchronization [18, 19], slide control [20], positivity analysis [21], and so on. It is said that the more information about time delay is used, the less conservative results will be obtained. In order to achieve this aim, some methods or techniques applied to improved Lyapunov functionals are proposed and used, such as slack variable method [1],stochastic approach [4, 5, 10], where the other results or methods can be found in the existing large references.

By investigating the most results on the system synthesis in the literature, it is seen that there were few references to consider the disordering problem. The motivation of disordering problem usually comes from the data transmitted through the shared communication networks. It is a phenomenon that the transmitted data arriving at the destination is usually out of order [22] and usually complicates its analysis and synthesis. During the past years, very few results were considered on this issue. Some new interesting and challenging problems could also been introduced. In reference [23], some LMI conditions were presented by exploiting a packet disordering compensation method. Based on transforming the underlying system into a discrete-time system with multi-step delays, the stability and H_∞ control problems of NCSs with packet disordering were considered in [24, 25], while some less conservative results were given in [26]. Recently, a kind of packet reordering method based on the average dwell-time method was proposed in [27]. By investigating such references, it will be seen that the considered problems and studied methods between these references and this paper are quite different. Firstly, the considered systems between these references and this paper are different. The originally considered systems of such references are ones without any time delay, while there is time delay in our considered systems. Secondly, the places of disordering happening are different. In the above references, the disordering only exists in system states transmitted through networks, while the disordering to be considered in this paper takes place between system states and control gains. A suitable model to describe such problems correctly should be established firstly. Thirdly, but not the last, even if a suitable model is presented, how to make the existence conditions of the desired controller with solvable forms is also necessary studied. To our best knowledge, very few results are available to design a disordered controller for delay systems. All the facts motivate the current research.

In this paper, the general stabilization for a class of stochastic delay systems closed a disordered controller is studied by a disorder-dependent approach. The main contributions of this paper are summarized as follows: 1) A kind of sta-

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bilizing controller experiencing a disorder between control gains and system states is proposed. Not only is the disorder described by the robust method but also its probability distribution are expressed by a Markov process with two modes; 2) Based on the established model of disordered controller, two different sufficient conditions for such a controller are presented with LMI forms, where a disorder-dependent approach in terms of depending on the Markov process is exploited; It has also shown by a numerical example that the conservatism of above conditions is not constant and should be considered on the concrete situations; 3) Because of all the results being LMIs, they are further extended to another general case that the TRM describing the disorder has uncertainties.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. $\mathscr{E}[\cdot]$ means the mathematical expectation of $[\cdot]$. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, diag $\{\cdots\}$ for a block-diagonal matrix, and $(M)^\star \triangleq M + M^T$.

2 FORMATTING INSTRUCTIONS

Consider a kind of stochastic delay systems described as

$$\begin{cases} dx(t) = [Ax(t) + A_\tau x(t-\tau) + Bu(t)]dt + [Cx(t) \\ \qquad\qquad + C_\tau x(t-\tau) + Du(t)]d\omega(t) \\ x(t) = \phi(t),\ t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, and $\omega(t)$ is an one-dimensional Brownian motion or Wiener process. Matrices A, A_τ, B, C, C_τ, and D are known matrices of compatible dimension. Time delay τ satisfies $\tau \geq 0$. $\phi(t)$ is a continuous function and defined from $[-\tau, 0]$ to $\mathbb{R}^n$.

As we know, the traditional state feedback controllers for delay systems are commonly as follows:

$$u(t) = Kx(t) \tag{2}$$

$$u(t) = K_\tau x(t-\tau) \tag{3}$$

$$u(t) = Kx(t) + K_\tau x(t-\tau) \tag{4}$$

where K and K_τ are control gains to be determined. It is said that controller (4) compared to (2) and (3) is more general and has some advantages in terms of being less conservatism. The main reason is both delay and non-delay states are taken into account. However, it is said that the action of controller (4) needs an assumption that the control gains and theirs related states should be available in a right sequence. Unfortunately, due to some practice constraints, this assumption may be very hard satisfied. In this paper, a kind of controller experiencing disordering phenomenon is proposed and described by

$$u(t) = \begin{cases} Kx(t) + K_\tau x(t-\tau), \text{ no disordering} \\ K_\tau x(t) + Kx(t-\tau), \text{ disordering occuring} \end{cases} \tag{5}$$

It is rewritten to be

$$u(t) = (\bar{K} + \Delta\bar{K}(\eta_t))x(t) + (\bar{K}_\tau + \Delta\bar{K}_\tau(\eta_t))x(t-\tau) \tag{6}$$

where $\bar{K} = \frac{1}{2}(K + K_\tau)$ and $\bar{K}_\tau = \frac{1}{2}(K_\tau - K)$.Particularly, the process $\{\eta_t, t \geq 0\}$ introduced here is a Markov process having two modes and assumed to take values in a finite set $\mathbb{S} \triangleq \{1, 2\}$. Its transition rate matrix (TRM) $\Pi \triangleq (\pi_{ij}) \in \mathbb{R}^{2\times 2}$ is given by

$$\Pr(\eta_{t+\Delta t} = j | \eta_t = i) = \begin{cases} \pi_{ij}\Delta t + o(\Delta t),\ i \neq j \\ 1 + \pi_{ii}\Delta t + o(\Delta t),\ i = j \end{cases} \tag{7}$$

where $\Delta t > 0, \pi_{ij} \geq 0$, if $i \neq j$, and $\pi_{ii} = -\sum_{j\neq i} \pi_{ij}$.Here, the gain fluctuations are selected to be

$$\Delta\bar{K}(\eta_t) = \begin{cases} \frac{1}{2}(K - K_\tau), \\ \text{if } \eta_t = 1 \text{ or no disordering} \\ \frac{1}{2}(K_\tau - K), \\ \text{if } \eta_t = 2 \text{ or disordering occuring} \end{cases} \tag{8}$$

$$\Delta\bar{K}_\tau(\eta_t) = \begin{cases} \frac{1}{2}(K + K_\tau),\ \text{if } \eta_t = 1 \\ \frac{1}{2}(3K - K_\tau),\ \text{if } \eta_t = 2 \end{cases} \tag{9}$$

Applying controller (6) to system (1), we get

$$\begin{cases} dx(t) = [(\bar{A} + B\Delta\bar{K}(\eta_t))x(t) \\ \qquad + (\bar{A}_\tau + B\Delta\bar{K}_\tau(\eta_t))x(t-\tau)]dt \\ \qquad + [(\bar{C} + D\Delta\bar{K}(\eta_t))x(t) \\ \qquad + (\bar{C}_\tau + D\Delta\bar{K}_\tau(\eta_t))x(t-\tau)]d\omega(t) \\ x(t) = \phi(t),\ \eta_t = \eta_0,\ \forall t \in [-\tau, 0] \end{cases} \tag{10}$$

where

$$\bar{A} = A + B\bar{K},\ \bar{A}_\tau = A_\tau + B\bar{K}_\tau$$
$$\bar{C} = C + D\bar{K},\ \bar{C}_\tau = C_\tau + D\bar{K}_\tau$$

In this paper, the gain fluctuations $\Delta\bar{K}(\eta_t)$ and $\Delta\bar{K}_\tau(\eta_t)$ with forms (8) and (9) satisfy

$$\Delta\bar{K}^T(\eta_t)\Delta\bar{K}(\eta_t) \leq W,\ \Delta\bar{K}_\tau^T(\eta_t)\Delta\bar{K}_\tau(\eta_t) \leq W_\tau(\eta_t) \tag{11}$$

where W and $W_\tau(\eta_t)$ are positive-definite matrix to be determined.

REMARK 1 *It is worth mentioning that different from the traditional stabilization methods of delay systems that no disorder occurs in controllers, controller (5) has a disordering phenomenon between control gains and system states. Based on the robust method, the controller with disorder is transformed into a controller with special uncertainties. Moreover, the probabilities of non-disorder and disorder happening are described by a Markov process with two operation modes. It is seen that the proposed model (6) with conditions (8), (9) and (11) is fundamental in the disorder-dependent approach.*

DEFINITION 1 *System (10) is said to be stochastically stable, if there exists a constant $M(x_0, \eta_0)$ such that*

$$\mathscr{E}\{\int_0^\infty \|x(t)\|^2 dt | x_0, \eta_0\} \leq M(x_0, \eta_0)$$