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Low-voltage improved accuracy Gaussian function generator with fourth-order approximation

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ABSTRACT

An original realization of a CMOS Gaussian function generator is presented. The proposed method is based on a new approximation function that is able to fourth-order match the Gaussian function. The current-mode operation of the circuit strongly reduces the technological-caused errors and the errors introduced by temperature variations, with the result of an important increasing of the accuracy for the squaring circuit that represents the functional core of the Gaussian generator (0.1%). Additionally, the bandwidth of the generator is increased as a result of its current-mode operation. Because of the utilization of the new fourth-order approximation function, the deviation from the ideal Gaussian function is smaller than 1 dB for an extended range of the input variable. The circuit is designed for implementing in 0.18 μm CMOS technology, its proposed architecture being compatible with a low-voltage operation (V_{DD}=1 V). The proposed Gaussian function generator based on the new approximation function allows to extend its capability of generating any continuous mathematical functions, this feature being obtained by changing the approximation function coefficients.

1. Introduction

Analog signal processing represents an important area of electronics, being developed computational structures for implementing a multitude of functions: signal amplifying [1–7], squaring [8–11] or square-rooting [12–14] structures, Euclidean distance [15,16], multiplier/divider [17–20] or exponential circuits [21–24]. Besides these circuit functions, the Gaussian function [25–41] is widely used in many domains such as neural networks, neural algorithms, neuro-fuzzy and classification applications, on-chip unsupervised learning, wavelet transform and pattern recognition. Back-propagation neural networks that use synapses based on Gaussian function presents a much better convergence comparing with neural networks having linear/multiplying synapses. The requirements for analog implementations of neural networks are mainly related to the necessity of reducing the power consumption and to the increasing of the circuits' speed for a real-time operation. In this context, portable and medical applications employing neural networks are usually implemented using analog circuits.

It exists in literature a large number of Gaussian circuits, their realizations being based on different principles. The circuits proposed in [25] implement the Gaussian function replacing the classical MOS active devices with FGMOS (Floating Gate MOS) transistors and use these devices as variable resistors for increasing the bandwidth of the conventional circuit. In [26] it was reported a programmable Gaussian generator, this function being carried out by using current sources. In [27] it was presented a mixed-signal CMOS integrated circuit implementation of a Gaussian function for neural/fuzzy hardware, the programmability of the generator being obtained by varying the reference voltages and the size of transistors in differential pairs. The Gaussian function generator presented in [28] combines the exponential characteristics of MOS transistors in weak inversion with the squaring characteristics in saturation. In [29], a fully-programmable analog circuit for Gaussian function generator using switched-current (SI) technology was developed, the programmability being implemented and controlled by the clock frequency and by the transconductance ratios of SI filters. It was presented in [30] a compact analog synapse cell, which is not biased in the subthreshold region for fully-parallel operation, the cell being able to approximate with reasonable accuracy the Gaussian function only in the ideal case. The previous reported Gaussian function generators present the disadvantages of an important dependence of circuits' performances on technological-caused errors and on temperature variations, this fact being responsible for reducing the overall structures' accuracy.

2. Theoretical analysis

The original proposed method for implementing the Gaussian function with improved accuracy is based on the utilization of a new approximation function that is able to high-order match the Gaussian function. In order to improve the frequency response of the circuit and to strongly decrease the error of approximating the

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Gaussian function, the current-mode operation will represent the specificity of the design. The independence of the output current of the proposed Gaussian function generator on technological parameters contributes to an additional increase of the computation accuracy.

2.1. Fourth-order approximation function

The reasons for choosing the following particular approximation function are mainly related to the possibilities of its facile implementations in CMOS technology using the squaring characteristic of MOS transistors biased in saturation region. The necessity of the exclusive use of this specific biasing of MOS active devices is imposed by the requirements for a good frequency response of the proposed Gaussian circuit. In the same context, the current-mode operation strongly improves the frequency response of the proposed circuits. As the simplest implementation in CMOS technology of a mathematical function is represented by the squaring characteristic, it is preferable from complexity motivations to build the approximation function having functional basis as this squaring characteristic.

The requirements for an extreme accuracy of the Gaussian function generation, correlated to the need for an extended dynamic output range for the proposed circuits impose a relatively high order of approximation for the mathematical function that approximates the Gaussian characteristic. The increasing of the order of approximation for this function strongly increases the complexity of the designed Gaussian function generator. In this context, it is obviously the necessity of realizing a tradeoff between the circuit complexity and its overall accuracy. From this point of view, an optimal choice that permits to obtain a very good accuracy and a relative large dynamic output range of the Gaussian circuit using a reasonable circuit complexity is based on a fourth-order approximation of the Gaussian function.

Based on the previous considerations, the original proposed fourth-order approximation function could be generally expressed as follows:

$$g(x) = \frac{b}{1+ax} + \frac{c}{1+x} + dx + e \tag{1}$$

a, b, c, d and e being the constant coefficients having the values imposed by the condition that $g(x)$ approximation function should match, in a fourth-order approximation, the Gaussian function:

$$f(x) = A\exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{2}$$

A and σ being the constants that define the amplitude and the width of the Gaussian function, respectively. The $g(x)$ function contains five coefficients because the fourth-order identity between two mathematical functions is equivalent with the identity between their first five Taylor series coefficients.

The expression of $g(x)$ function that fourth-order approximates the particular $f(x) = \exp(-x^2)$ Gaussian function can be obtained for $A=1$ and $\sigma = 1/\sqrt{2}$, resulting

$$g(x) = -\frac{8}{1+(x/2)} + \frac{1}{1+x} - 3x + 8 \tag{3}$$

The superior-order Taylor series of the approximation function has the following expression:

$$g(x) = (b+c+e) + (d-c-ab)x + (c+a^2b)x^2 \\ -(c+a^3b)x^3 + (c+a^4b)x^4 - (c+a^5b)x^5 + \cdots \tag{4}$$

while the expansion in the same series of the Gaussian function can be expressed as follows:

$$f(x) = 1 - \frac{x^2}{2\sigma^2} + \frac{x^4}{8\sigma^4} - \frac{3x^6}{80\sigma^6} + \cdots \tag{5}$$

Because $f(x)$ is an even-order function, all the odd-order terms from its series expansion are zero. The fourth-order identity between the previous functions is equivalent with the necessity of simultaneously fulfilling the following five mathematical relations:

$$b+c+e = 1 \tag{6}$$

$$d-c-ab = 0 \tag{7}$$

$$c+a^2b = -\frac{1}{2\sigma^2} \tag{8}$$

$$c+a^3b = 0 \tag{9}$$

$$c+a^4b = \frac{1}{8\sigma^4} \tag{10}$$

Solving this system, it results the following values for $a,\ldots,e$ coefficients from the general expression of the proposed $g(x)$ approximation function

$$a = \frac{1}{4\sigma^2} \tag{11}$$

$$b = \frac{32\sigma^4}{1-4\sigma^2} \tag{12}$$

$$c = \frac{1}{2\sigma^2(4\sigma^2-1)} \tag{13}$$

$$d = \frac{16\sigma^4+1}{2\sigma^2(1-4\sigma^2)} \tag{14}$$

and

$$e = \frac{64\sigma^6-1}{2\sigma^2(4\sigma^2-1)} + 1 \tag{15}$$

As a result, the $g(x)$ function can be expressed replacing (11)–(15) in (1).

As the fifth-order term of the Taylor series expansion is zero, the approximation error is mainly given by the sixth-order term from the same expansion:

$$\varepsilon_{f(x)}^{g(x)} \cong \frac{3}{80\sigma^6\exp(-(x^2/2\sigma^2))} = \frac{3}{10\exp(-x^2)} \tag{16}$$

2.2. Implementation of the Gaussian function generator

The block diagram of the Gaussian function generator, based on the original proposed approximation function (3), is presented in Fig. 1. I_{IN} represents the input current, I_O is a constant reference current, while I_{OUT} signifies the output current that must be made to be proportional with the Gaussian function. The value of σ width from the Gaussian function (2), approximated by the circuit having the block diagram presented in Fig. 1, can be set by

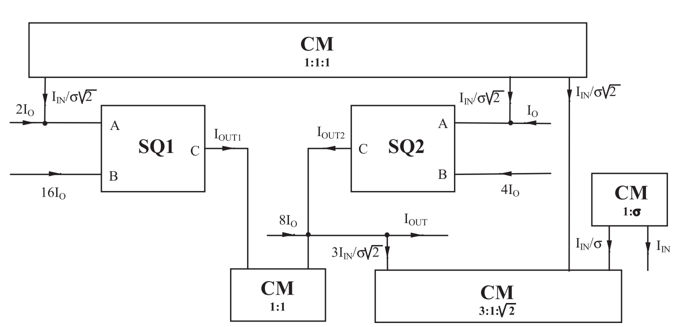

Fig. 1. The block diagram of the Gaussian function generator.