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The influence of vertical deflection of the supports in modeling squeeze film damping in torsional micromirrors

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ABSTRACT

The objective of this work is to create an analytical framework to study the problem of squeezed film damping in micromirrors considering the bending of the supporting torsion microbeams. Using mathematical and physical justifications, nonlinear Reynolds equation governing the behavior of the squeezed gas underneath the mirror is linearized. The resulting linearized equation is then non-dimensionalized and analytically solved for two cases of the infinitesimal and finite tilting angle of the mirror. The obtained pressure distribution from the solution of the Reynolds equation is then utilized for finding the squeezed film damping force and torque applied to the mirror. The results show that in the case of the infinitesimal tilting angle, the squeezed film damping can be modeled with a linear viscous damping in both torsional and lateral directions. It is also shown that when the mirror's rotation angle is small, with increasing the length of the mirror, the damping force and damping torque are increased. For the case of the finite tilting angle it was observed that the applied damping torque highly depends on the tilting angle of the mirror as well as the ratio of its vertical to angular velocity and as a result the effect of the vertical velocity of the mirror on the squeezed film damping force and torque applied to the mirror cannot be simply neglected. It is expected that the qualitative and quantitative knowledge resulting from this effort will ultimately allow the analysis, optimization, and synthesis of micromirrors for improved dynamic performance.

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1. Introduction

MEMS devices use parallel plate capacitors in which one plate is actuated electrostatically and its movement is detected with capacitive changing. In order to increase the excitation performance as well as the detection sensitivity, the distance between capacitive plates is minimized and the area of the electrodes is maximized. In such a condition, squeeze film damping becomes the most important energy loss mechanism in MEMS. In fact squeeze film damping is the result of massive movement of the trapped gas molecules to the space between electrodes which is opposed by the gas viscosity. This mechanism produces some kind of pressure distribution underneath the plate which can act like a damping force or like a spring force.

Currently there are two approaches for modeling the damping mechanism of the microresonators in the rare gas ambient. The first approach presented by Veijola et al. [1] suggests an effective coefficient of viscosity in which an approximated viscosity coefficient depends on the gas pressure via the Knudsen number of the system. Then by solving the Reynolds equation which governs the squeeze film damping phenomenon and utilizing this empirical coefficient in the solution the damping effect can be predicted for different ambient pressures [2]. An alternative approach presented by Christian [3], Bao et al. [4] and Hutcherson and Ye [5] is based on free molecular dynamic models developed for a plate vibrating in normal direction to a nearby stationary wall [2]. The mentioned model is based on momentum transfer rate from the vibrating plate to the surrounding gas due to collisions of gas molecules with the plate.

In recent years, more and more torsion micro-mirrors have been used in a variety of MEMS devices, such as optical displays, light modulator and optical switches. As the squeeze film damping is the key factor to the dynamic performance of the mirror, it has been investigated extensively in recent years. In micromirrors, since the gap distance and the moving speed of the plate are not uniform, the analysis of the squeeze film air damping of torsion mirrors becomes more difficult than that of a parallel plate actuator [6].Chang et al. [7] modeled the squeeze film damping using the so called modified molecular gas film lubrication equation with the coupling effects of surface roughness and gas rarefaction. Hao et al. [8] provided analytical expressions for

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damping pressure of a rectangular mirror at its balanced position and discussed the influence of design parameters. Pan et al. [9] presents analytical solutions for the effect of squeeze film damping, on a MEMS torsion mirror using Fourier series solution and the double sine series solution under the assumption of small displacements. For the purpose of verification, they also used a numerical finite difference scheme to obtain squeeze film damping torque and used their analytical and numerical formula to simulate the dynamic response of the micromirror and found out that the two approaches yield almost the same outcomes. They also performed experimental measurements and obtained results that were consistent with those obtained from the analytical and numerical damping models. Excluding the tilting angle which was considered to be small, the main problem of this work was that Pan et al. [9] assumes the excitation to be harmonic function of time and so the response would also be a single harmonic function of time, while in real situations where the mirror is actuated electrostatically, the excitation is not only a function of time, but also a strongly nonlinear function of the response, i.e. tilting angle of the mirror. So one cannot assume the response to be a simple harmonic time function. Minikes et al. [2] adapted the squeeze film model with artificial viscosity and the molecular dynamics model for the case of a torsion mirror under a wide range of vacuum levels. They employed the Green's function technique to solve the linearized Reynolds equation. Their method was based on the assumption that the mirror response is a single harmonic function of time which is valid only when the excitation is a single harmonic time function. This kind of excitations does not hold for the electrostatically actuated micromirrors where the excitation is a strongly nonlinear function of the response. Bao et al. [10] proposed an analytical model for calculating the squeeze film air damping of a rectangular torsion mirror at finite normalized tilting angles. Based on Reynolds equation they found damping pressure, damping torque and the coefficient of damping torque as functions of tilting angle and aspect ratio of the micromirror for the two cases of infinite long torsion mirror and rectangular micromirror with finite aspect ratio. In the most general case where the micromirror aspect ratio is finite and its tilting angle is not infinitesimal, they obtained an infinite series for the coefficient of damping torque where the coefficients of the series were complicated integrals with integrands which were explicit functions of normalized tilting angles. They assumed that the torsion beams supporting the mirror do not undergo bending, while there has been extensive studies [11–13] which show that the bending of torsion beams in micromirrors have significant effect on the statical and dynamical behavior of micromirrors.

In this study, the squeezed film damping model for mirrors presented by Bao et al. [10] is extended to analytically solve the problem of squeeze film damping in micromirrors considering the bending of torsion beams and analytic expressions is suggested for the damping force and damping torque in both cases of infinitesimal and finite tilting angles.

2. Problem formulation

Fig. 1 shows schematic view of a torsion micromirror. In electrostatically actuated torsional micromirrors suspended over a large gap of air, the tilting angle may be considered finite and as a result the thickness of the air gap can be assumed to be approximately constant. In such a condition, squeeze film air damping of micromirror is approximately governed by the Reynolds equation [6]. For very small geometries like micromirrors, where inertial effects of the squeezed air is negligible with respect to its viscous effect, the Reynolds equation reduces to [6].

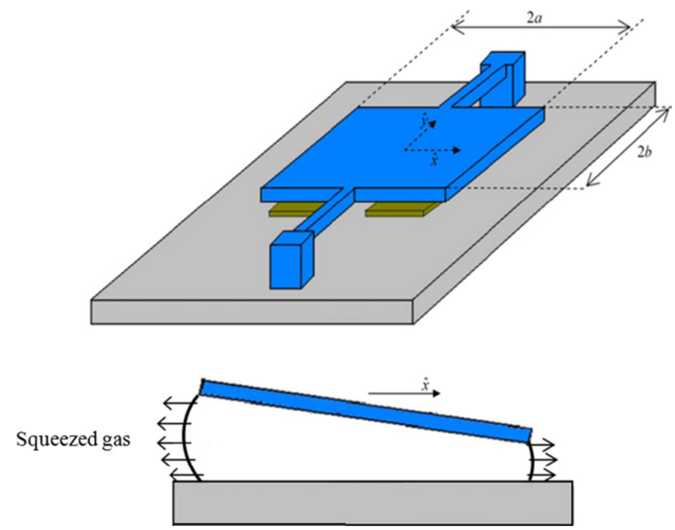

Fig. 1. Schematic view of micromirror under the effect of squeeze film damping.

$$\frac{\partial}{\partial \hat{x}}\left(\rho\frac{h^3}{\mu}\frac{\partial P}{\partial \hat{x}}\right)+\frac{\partial}{\partial \hat{y}}\left(\rho\frac{h^3}{\mu}\frac{\partial P}{\partial \hat{y}}\right)=12\frac{\partial(h\rho)}{\partial t} \tag{1}$$

where ρ is the fluid density, h is the fluid thickness at point $(\hat{x},\hat{y})$, μ is the fluid viscosity, P is the fluid pressure and t is the time. Under the isothermal condition which is the condition which usually arises in MEMS devices, the gas density ρ is directly proportional to its pressure P. So Eq. (1) can be rewritten in the form:

$$\frac{\partial}{\partial \hat{x}}\left(P\frac{h^3}{\mu}\frac{\partial P}{\partial \hat{x}}\right)+\frac{\partial}{\partial \hat{y}}\left(P\frac{h^3}{\mu}\frac{\partial P}{\partial \hat{y}}\right)=12\frac{\partial(hP)}{\partial t} \tag{2}$$

It has to be noticed that the pressure P is composed of two parts $P=p_a+p$ where p_a is the ambient pressure and p is the relative pressure which is due to the squeezed film effect. For small displacement of the plate around its balance position ($\Delta h \ll h_0$ and $p \ll p_a$), Eq. (2) can be linearized as

$$\frac{\partial^2 p}{\partial \hat{x}^2}+\frac{\partial^2 p}{\partial \hat{y}^2}-\frac{12\mu}{p_a h^2}\frac{\partial p}{\partial t}=\frac{12\mu}{h^3}\frac{dh}{dt} \tag{3}$$

when $((12\mu/p_a h^2)(\partial p/\partial t))/((12\mu/h^3)(dh/dt)) \ll 1$ or $\Delta p/p_a \ll \Delta h/h$, the gas is not appreciably compressed [6]. The mentioned condition is referred as incompressible gas condition, under which the Eq. (3) is reduced to [10]:

$$\frac{\partial^2 p}{\partial \hat{x}^2}+\frac{\partial^2 p}{\partial \hat{y}^2}=\frac{12\mu}{h^3}\frac{dh}{dt} \tag{4}$$

In electrostatically actuated micromirror, the thickness of the fluid gap is a linear function of the position $\hat{x}$:

$$h=h_0-\theta\hat{x} \tag{5}$$

where h_0 is the initial gap between the mirror plate and the underneath substrate. Assuming the torsion beams supporting the mirror bend under the effect of the actuation force or squeezed film pressure which is a situation usually raised in micromirrors, h_0 would not be constant with respect to time. In such a condition, one can conclude:

$$\frac{dh}{dt}=\dot{h}_0-\dot{\theta}\hat{x} \tag{6}$$

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