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# Interaction force between ultrathin multilayer films induced by quantum fluctuations

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ABSTRACT

The van der Waals force between dielectric plates comprising ultrathin conducting films was calculated by employing two models. In the first model, the ultrathin dielectric film was modeled as a dielectric plate of a small thickness, and the Lifshitz theory was applied. In the second model, an ultrathin dielectric film was modeled as a plasma sheet, which is an infinitesimally thin fluid carrying mass and charge. For each model, the interacting force was considered to be a function of the separation gap between the films, and the difference in the force-distance relationships of two was discussed.



## 1. Introduction

The field of synthesizing novel ultrathin materials, such as graphene, has developed rapidly. Moreover, in future, the ultrathin materials are expected to be combined in order to manufacture van der Waals heterostructures [1,2]. The van der Waals force, which is arisen by quantum fluctuations is an important forces combining ultrathin materials. However, the general method for computing the van der Waals force between multilayer films comprising ultrathin films is not yet clear, because there are two difficulties in the computation. The first problem is that the optical properties of materials in ultrathin films are different from when they are in bulk. For example, when calculating the van der Waals force between graphene, the Dirac model is required, which is based on quantum field theory. The second problem is that the electromagnetic field between two films depends on the presence of other films; thus, the van der Waals force depends on the number of films, each of which comprise multilayers. In this study, we focus on the second problem.

The van der Waals force between conducting plates arises from a change in the quantized electromagnetic field in the presence of boundary surfaces, and the force rapidly increases as the separation distance between the conducting plates decreases [3–5]. Parallel configurations of plates that have a small gap between them can often be seen in multilayer films; for example, graphite is multilayer films comprising of grapheme with a gap as small as 0.335 nm. If conducting plates perfectly reflect electromagnetic waves of an arbitrary frequency, the Casimir force per unit area between two plates separated by a vacuum gap $a$ is expressed as $-1.3$ GPa/$a^4$ ($a$ in nm). Thus, the strength of the pressure can reach 1.3 GPa when the separation distance decreases down to 1 nm.

In practice, however, the force between actual multilayer films decreases significantly due to the finite permittivity and finite thickness of conducting films. This is because the van der Waals force strongly depends on the reflection coefficient, which is mainly determined by the permittivity and thickness of the conducting plates in a system. Unlike perfectly conducting plates, the permittivities of actual materials decrease as the frequency increases in the high-frequency regime. For example, the force between dielectric plates obeying the plasma model with $10^{16}$ Hz is 0.02% of the Casimir force between perfectly conductive plates at 1 nm separation gap. In addition, in most cases the van der Waals force between two dielectric plates decreases as the thickness of the plates decreases. The film thickness of a multilayer film is, in general, very small; it is a factor that make the van der Waals force smaller. However, a multilayer structure of a dielectric film can increase the van der Waals force between the films.

The van der Waals interaction between multilayers has been studied [5–12], and various formulas have been presented for different types of multilayers with finite thicknesses. However, the formulas are often written as recurrence formulas. In this study, we

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explicitly express the van der Waals potential energy as a summation that does not use any recurrence formulas. Recently, Khusnutdinov, Kashapov, and Woods [13,14] studied the Casimir effect for a stack of infinitely thin conductive plates, and they considered the force acting on a particular plane in the stack. Using their results, they were able to discuss the interaction between this particular plane in multilayers and other layers. Unlike the force acting on a particular plane, we are interested in the adhesion force between two plates comprising alternating thin conducting-insulating layers, which is important in the synthesis of van der Waals heterostructures. In particular, we will examine how the adhesion force is affected by the number of layers in a film.

This paper is structured as follows. In Section 2, we extend the Lifshitz formula for the case of two dielectric plates to calculate the van der Waals force between the alternating layers, that comprise two types of dielectric plates, based on the studies of the van der Waals force between multilayers [5–11]. In this calculation, the dispersion relation of the electromagnetic field, i.e. the relationship between the wavenumber and the frequency, is of primary importance, and is explicitly expressed for an arbitrary number of layers. We are interested in understanding the van der Waals force between alternating multilayers comprising thin layers that are almost of the same thickness. Thus, we fix the ratio of the film thickness and the separation, and consider the dependence of the van der Waals force on the separation distance. In Section 3, we introduce a plasma sheet model [15] to describe very thin conducting layer. The van der Waals force between individual plasma sheets is calculated using the transmittance of the sheets, and the dependence of the van der Waals force on the number of plasma sheets is considered. Finally, in Section 4, we discuss the difference between the results obtained using the two methods outlined in Sections 2 and 3.

## 2. Van der Waals force between dielectric multilayers

### 2.1. Lifshitz-type formula for a dielectric multilayer

We consider alternating multilayers comprising two types of dielectric plates-conducting and insulating. However, the formulation derived below is valid for an arbitrary number of dielectric multilayers if their permittivities depend only upon the frequency of the applied electric field. As shown in Fig. 1, the conducting plate and insulating plate have dielectric permittivities $\epsilon_C(\omega)$ and $\epsilon_I(\omega)$, respectively, which depend on the frequency, $\omega$, of an applied electric field. The thicknesses of the plates with the dielectric permittivities $\epsilon_C(\omega)$ and $\epsilon_I(\omega)$ are $d_C$ and $d_I$, respectively, except for the outermost layers. We assume that the dielectric permittivities of the outermost layers (i.e. the substrate) are $\epsilon_C(\omega)$ and that their thicknesses are infinite. Furthermore, we impose a condition that the number of layers $N$ is $n+1$, where $n$ is a positive integer. If this condition is satisfied, then the center of the layer is always occupied by a dielectric plate with permittivity $\epsilon_I(\omega)$. Each layer is labeled by an integer, $j = 0, 1,.., n$. We calculate the van der Waals force between the layers labeled by $j = n/2 - 1$ and $n/2 + 1$, which are on either side of a middle dielectric layer, labeled by $j = n/2$. The thicknesses of the dielectric layers are fixed to $d_C$. The thicknesses of the insulating layers are fixed to $d_I$ except for those of the thickness of middle dielectric layer $a$ and the layers contacting on the outermost layers $L$ (see Fig. 1). Thus, the van der Waals force can be expressed as a function of four parameters: $d_I$, $d_C$, $a$ and $L$. When the distance between the conducting layers and the substrate $L$ is infinity, the influence of the van der Waals force on the extra set of boundary conditions at the substrate disappears.

The van der Waals force is calculated in terms of the zero-point energy of a quantized field in the multilayer. We consider a monochromatic electromagnetic field in the multilayer. The complete orthonormal set of solutions to the Maxwell equations for

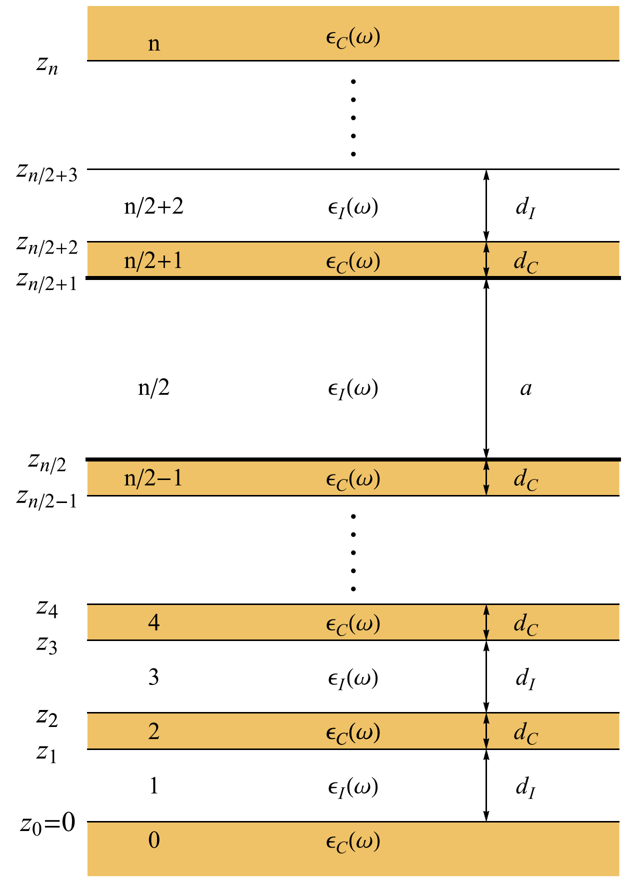


**Fig. 1.** Alternating multilayers comprising conducting layers of thickness $d_C$ (the area with the hatching) and insulating layer of thickness $d_I$ (the area without hatching). The thickness of the middle layer located between $z_{n/2}$ and $z_{n/2+1}$ only is $a$. The thickness of the outer-most layer is infinite. The distance between the substrates and the nearest neighbor conducting layer is $L$.

the electric field between $z_{j-1}$ and $z_j$, which respectively have the dielectric permittivities $\epsilon_j(\omega) \in \{\epsilon_C(\omega), \epsilon_I(\omega)\}$ can be expressed as

$$\mathbf{E}_j(\mathbf{r}, \mathbf{k}_\perp, \sigma) = \mathbf{e}_{j,\sigma}(z, k_\perp) e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp}. \tag{1}$$

Here, $\mathbf{r} = (x, y, z) = (r_\perp, z)$, $k_\perp$ is a wavenumber parallel to the dielectric plate, and $\sigma \in$ {TM,TE} denotes the transverse-magnetic (TM) and transverse-electric (TE) polarization modes of the electromagnetic field. The $z$-component of the vector $\mathbf{e}_{j,\sigma}(z, k_\perp)$ for the $j$th plate can be represented by

$$e_{j,\sigma}(z, k_\perp) = A_{j,\sigma} e^{q_j z} + B_{j,\sigma} e^{-q_j z}, \tag{2}$$

where $q_j \equiv q_j(k_\perp, \omega) = \sqrt{k_\perp^2 - \epsilon_j(\omega)\omega^2/c^2}$. Assuming an exponential decrease for $z < 0$ and $z > z_n$, then the coefficient $B_{0,\sigma}$ and $A_{n,\sigma}$ must be zero. The electromagnetic field of the TM mode in the multilayer must satisfy the following boundary condition at the $z$-coordinate $z_j$ of the $j$th dielectric layer:

$$\epsilon_{j-1}(\omega) e_{j-1,\mathrm{TM}}(z_j, k_\perp) = \epsilon_j(\omega) e_{j,\mathrm{TM}}(z_j, k_\perp), \tag{3}$$

$$e'_{j-1,\mathrm{TM}}(z_j, k_\perp) = e'_{j,\mathrm{TM}}(z_j, k_\perp), \tag{4}$$