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Spin polarisation with electron Bessel beams

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ABSTRACT

The theoretical possibility to use an electron microscope as a spin polarizer is studied. It turns out that a Bessel beam passing a standard magnetic objective lens is intrinsically spin polarized when post-selected on-axis. In the limit of infinitely small detectors, the spin polarisation tends to 100 %. Increasing the detector size, the polarisation decreases rapidly, dropping below 10^{-4} for standard settings of medium voltage microscopes. For extremely low voltages, the Figure of Merit increases by two orders of magnitude, approaching that of existing Mott detectors. Our findings may lead to new desings of spin filters, an attractive option in view of its inherent combination with the electron microscope, especially at low voltage.

1. Introduction

After the prediction [1] and the experimental demonstration [2,3] of free electrons carrying orbital angular momentum (OAM), these vortex beams–as they are called–are increasingly attracting interest. They are characterized by a spiralling wavefront, similar to optical vortices [4]. In parallel to their OAM, they carry magnetic moment that is independent of their spin polarisation. Their potential ranges from the study of Landau states in an interaction-free environment [5], over probing chiral specimens with elastic or inelastic scattering on the nanoscale, to manipulation of nanoparticles, clusters and molecules, exploiting the magnetic interaction and the transfer of angular momentum [6]. Bliokh et al. [7] discovered an unexpected intrinsic spin-orbit coupling (SOC) in relativistic vortex electrons, and proposed to use this effect in a spin filter.

Despite the statement of Bohr and Pauli that Stern-Gerlach based spin separation for electrons cannot work [8], it has been argued that spin separation or filtering of electrons is possible in particular geometries [9,10]. The argument has been debated, see e.g. [11], and it seems that the effect exists but is too small to be exploited with present day technology. In later papers, interesting alternatives relying on inhomogeneous magnetic fields with cylindrical symmetry [12,13], or on SOC in an electron transparent medium [14] were discussed. But as of now, no Stern-Gerlach design of a spin polarizer for free electrons was successful.

In an elegant derivation Karimi et al. [15] have shown that crossed electric and magnetic quadrupole fields correspond to an optical q-plate with $q = -1$. In combination with electron vortex beams, this opens the possibility to couple the spin of free electrons to the spatial degree of freedom, and so design a spin filter [16].

The proposal is an analogue to the spin-to-orbital momentum conversion (STOC) of optical beams [17,18]. The essential driving agent for the STOC process is a non-vanishing Berry connection [1]. This translates either into the connection $\mathcal{A}(\mathbf{p})$ in momentum space, or in the presence of a magnetic field into the vector potential $\mathbf{A}(\mathbf{r})$ in real space. $\mathbf{A}(\mathbf{r})$ with cylindrical symmetry over the propagation axis is equivalent to an optical q-plate with $q = 1$. Such a field can be used as a STOC device quite similar to the optics case because the total angular momentum $L + S$ is a constant of motion.[1] This configuration is encountered in a magnetic round lens used in electron microscopes. Thus, it seems that electron microscopes are intrinsic spin polarizers for electron vortex beams.

Instead of solving the Dirac equation in the inhomogeneous lens field, which is a complicated numerical problem we aim at an estimate of the order of magnitude of the STOC effect to be

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[1] Which is also evident from the radial symmetry of the in-plane component of the magnetic field. Note that L is the angular momentum in the corotating Larmor frame. The fact that the *mechanical* total angular momentum $\mathcal{L} + S$ in the laboratory frame is not conserved has no consequence on the STOC process.

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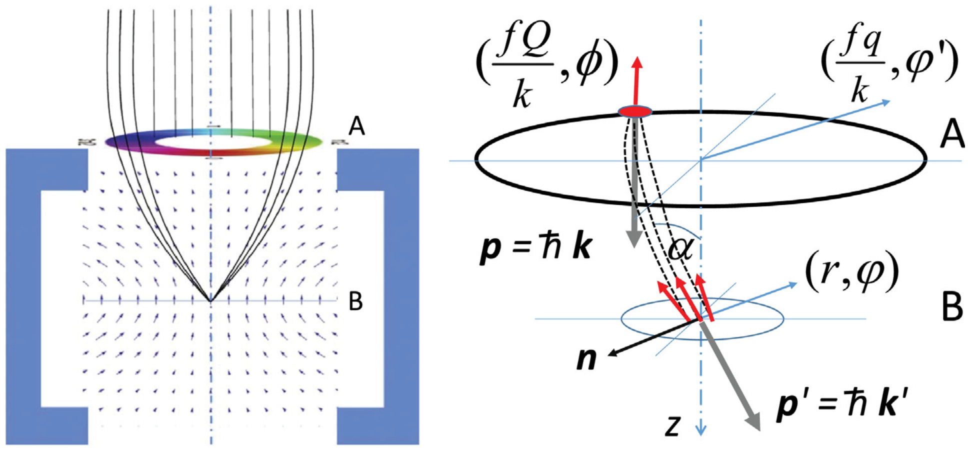

Fig. 1. Schematic of the proposed geometry. Left: A plane wave incident in the front focal plane of a magnetic round lens is limited by an annular aperture selecting a narrow range of lateral momenta. A spiral phase plate (or a similar vortex generating device) imprints an azimuthal phase ramp ϕ thus creating a vortex beam with OAM=1. The phase is indicated by the hue colorscale along the ring aperture. The image plane contains the Fourier transform, a Bessel beam J_1. The yoke and the magnetic field are indicated. Right: Cylindical coordinates in planes A and B (blue arrows). The pointlike source (red dot) with spatial coordinates (fQ/k, ϕ), where f is the focal width and k is the wave number, on the ring aperture (the integrand in Eq. (1)) creates a plane wave with wave vector $\mathbf{k}'$ in plane B. In the classical description, the momentum $\mathbf{p} = \hbar\mathbf{k}$ of the particle in A is tilted by the Lorentz force of the magnetic lens field to $\mathbf{p}' = \hbar\mathbf{k}'$ at B (grey arrows). The spin vector of a hypothetical spin polarized electron (red arrows) performs a precession in the magnetic field when the outgoing wave propagates to plane B, conserving helicity. Three Bohmian trajectories are shown (dashed lines) together with the r-dependent spins, slightly deviating from the tilt or convergence half angle α over the unit vector $\mathbf{n}$. Vectors are not to scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

expected under reasonable conditions. For an intuitive understanding of the underlying principle, we shall make use of the concept of Bohmian trajectories [19,20].

2. Theory and results

We assume a plane wave parallel to the optic axis, we^{ikz}, incident in the front focal plane A of a round magnetic lens, as sketched in Fig. 1. $w = (a_+, a_-)^T$ is the Pauli spinor in the eigenbasis of the Pauli matrix σ_z, and the quantisation axis z is the optic axis.[2] The coefficients $a_\pm$ are normalized as $|a_+|^2+|a_-|^2 = 1$. The spinor is assumed to be independent of the spatial coordinates of the plane wave, i.e. the plane wave has unique spin polarisation.

The wave function is prepared by a ring aperture imprinting an azimuthal phase ramp of 2π. This can be achieved by a spiral phase plate, a holographic mask, a thin magnetic needle [21], or any other method suitable to produce beams with topological charge. For the moment, we assume an infinitesimally narrow ring of radius R which in paraxial approximation equals $fQ/k = \alpha f$. f is the focal width of the lens, and Q is the corresponding lateral wave number when we consider plane A as the (front) focal plane of a lens representing reciprocal variables, as usual in electron microscopy notation. This ring prepares the electron in plane A located at position z=0.

Then the wave function after passing the ring aperture is

$|\Psi\rangle = (a_+|\uparrow\rangle + a_-|\downarrow\rangle) \otimes |\psi_A\rangle.$

In terms of cylindrical coordinates (q, φ'), ψ_A is an azimuthal superposition of point sources on a ring with radius Q,

$$\psi_A(q,\varphi') = \frac{1}{\sqrt{2\pi Q}}\int_0^{2\pi} \delta(q-Q)\delta(\varphi'-\phi)e^{i\phi}d\phi. \tag{1}$$

In the magnetic field of the lens, the wave propagates from plane A to plane B.

The point (Q, φ) can be interpreted as the source of a bundle of Bohmian trajectories that cross the optic axis close to plane B, with convergence half angle α. Since the angle between spin and momentum remains constant in the magnetic field, the spin direction in B depends on the position, as sketched in Fig. 1. In the narrow radial range of a few nm centered at the optic axis that interests us later, the deviation of the spin directions from α is negligible, so we may consider α as independent of the radial distance r in plane B.

Thus, on arrival at plane B the initial spin will have been rotated by $\approx\ \alpha$ over some direction $\mathbf{n}$ on the Bloch sphere. The operator for the spinor rotation is

$$R_{\mathbf{n}}(\alpha) = e^{-i\frac{\alpha}{2}\mathbf{n}\cdot\boldsymbol{\sigma}}, \tag{2}$$

with the Pauli matrices $\boldsymbol{\sigma}$.

The transfer of the *spatial* part, i.e. a point source $\psi_{Q,\phi}$ from the front focal plane A to the object plane B can be described by a 2D Fourier transform. Combining this with the spinor rotation, the point source at (Q, ϕ) in plane A has evolved into

$$\begin{aligned}\tilde{\psi}_{Q,\phi}(r,\varphi) &= R_{\mathbf{n}}(\alpha)w\int q\ dq\ d\varphi'\ e^{i\mathbf{q}\cdot\mathbf{r}}\psi_{Q,\phi}(q,\varphi')\\ &= R_{\mathbf{n}}(\alpha)we^{iQr\cos(\phi-\varphi)}e^{i\phi}e^{ikz}\end{aligned} \tag{3}$$

in plane B. As expected, the spatial part is a plane wave with wave vector $\mathbf{k}' = (Q\cos\phi, Q\sin\phi, k)$ and a rotated spinor.[3]

The total wave function ψ_B in plane B is the superposition (*i.e.* azimuthal integral) over all partial plane waves Eq. (3),

$$\psi_B(r,\varphi) = \tilde{\psi}_A = \frac{1}{2\pi}\int_0^{2\pi} d\phi\tilde{\psi}_{Q,\phi}(r,\varphi) := Tw\psi_A \tag{4}$$

The transfer matrix T defined here is derived in the methods section:

$$T = ie^{ikz}\begin{pmatrix} J_1(Qr)e^{i\varphi}\left(\cos\frac{\alpha}{2} - i\sin\frac{\alpha}{2}\cos\theta\right) & -J_0(Qr)\sin\frac{\alpha}{2}\sin\theta \\ J_2(Qr)e^{2i\varphi}\sin\frac{\alpha}{2}\sin\theta & J_1(Qr)e^{i\varphi}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\cos\theta\right)\end{pmatrix}. \tag{5}$$

Ignoring the common phase factor, the special case of a spin up (spin down) state at plane A evolves into a state

$$|\uparrow\rangle\otimes\psi_A\to T|\uparrow\rangle = J_1(Qr)e^{i\varphi}\cos\frac{\alpha}{2}|\uparrow\rangle + J_2(Qr)e^{2i\varphi}\sin\frac{\alpha}{2}|\downarrow\rangle \tag{6a}$$

[2] Any other basis would also work.

[3] In paraxial approximation. The azimuth φ refers to the corotating Larmor system, that takes into account the image rotation in the lens [22].