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2nd-order optical model of the isotopic dependence of heavy ion absorption cross sections for radiation transport studies

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ABSTRACT

Heavy ion absorption cross sections play an important role in radiation transport codes used in risk assessment and for shielding studies of galactic cosmic ray (GCR) exposures. Due to the GCR primary nuclei composition and nuclear fragmentation leading to secondary nuclei heavy ions of charge number, Z with $3 \le Z \ge 28$ and mass numbers, A with $6 \le A \ge 60$ representing about 190 isotopes occur in GCR transport calculations. In this report we describe methods for developing a data-base of isotopic dependent heavy ion absorption cross sections for interactions. Calculations of a 2nd-order optical model solution to coupled-channel solutions to the Eikonal form of the nucleus-nucleus scattering amplitude are compared to 1st-order optical model solutions. The 2nd-order model takes into account two-body correlations in the projectile and target ground-states, which are ignored in the 1st-order optical model. Parameter free predictions are described using one-body and two-body ground state form factors for the isotopes considered and the free nucleon-nucleon scattering amplitude. Root mean square (RMS) matter radii for protons and neutrons are taken from electron and muon scattering data and nuclear structure models. We report on extensive comparisons to experimental data for energy-dependent absorption cross sections for over 100 isotopes of elements from Li to Fe interacting with carbon and aluminum targets. Agreement between model and experiments are generally within 10% for the 1st-order optical model and improved to less than 5% in the 2nd-order optical model in the majority of comparisons. Overall the 2nd-order optical model leads to a reduction in absorption compared to the 1st-order optical model for heavy ion interactions, which influences estimates of nuclear matter radii.

1. Introduction

Nuclear absorption cross sections for projectile energies up to about 50,000 MeV/u are used in the description of radiation transport for applications of space radiation protection and shielding, and at lower energies (< 500 MeV/u) in Hadron cancer therapy. For galactic cosmic ray (GCR) studies, heavy ions up to nickel ($Z = 28$) are considered for a variety of spacecraft shielding and tissue constituent atoms with a large number of isotopes occurring due to nuclear fragmentation and the isotopic composition of the primary GCR particles [1]. In deterministic transport models based on a Boltzmann equation, the absorption cross-section describes the overall attenuation of beam particles, while in stochastic Monte-Carlo transport models the absorption cross section determines the probability of nuclear reaction occurrence in particle propagation through materials [2–4]. Thus the absorption cross section is the leading order descriptor of high-energy charged particle transport after the Coulomb interaction, which is evaluated the continuous slowing down or perhaps including descriptions of energy-loss straggling. The absorption cross section also places an important constraints on nuclear fragmentation cross sections.

Because of the strong nature of the nuclear force, non-perturbative methods are needed to describe nuclear absorption. The multiple-scattering series of Watson [5] for proton nucleus scattering was extended to the nucleus-nucleus case by Wilson [6,7] and forms the basis for considering the elastic channel for determination of the total absorption cross section, and the development of models for the various inelastic channels. The coupled-channels approach to the nucleus-nucleus scattering formalism provides an interesting alternative to the Glauber model [8], while a Lippman-Schwinger equation for an identical double-folding potential can be used to consider the accuracy of the Eikonal approximation [9]. The coupled-channel solutions in the Eikonal approximation presents a matrix approach to consider systematic solutions for elastic and inelastic channels [10,11]. In previous work we developed a 2nd-order optical solution to Eikonal coupled-channels model of the nucleus-nucleus scattering amplitude [11,12]. The 2nd-order model considers two-body correlations in the projectile

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and target ground-states. Empirical models based on matter radii and simple models of nuclear densities can also be used to fit absorption cross sections, however they have not considered the isotopic dependence of absorption cross sections in the past [13,14].

In this work we compare the 2nd-order model to the more widely used 1st-order optical model, while focusing on absorption cross sections for a wide range of isotopes that appear in GCR transport calculations. For mass number, A > 16 we use the Woods-Saxon single particle density and related form factors with proton and neutron dependent half-density radius and surface diffuseness parameters based on a RMS proton charge radii from electron and muon scattering experiments [15] with adjustments for neutrons based on a recent nuclear structure analysis using the Skyrme potential [16]. For lighter nuclei we use the modified Harmonic oscillator (MHO) model. Model predictions are compared to absorption measurements [17–20] for over 100 isotopes for elements from Li to Fe on C and Al targets for energies near 1000 MeV/n.

2. Methods

In the 2nd-order solution to the Eikonal form of optical model coupled-channels equations for the heavy ion scattering [10–12], the elastic amplitude, $f(q)$ is given by

$$f(\vec{q}) = \frac{-ik}{2\pi}\int d\vec{b}\ \exp(-i\vec{q}\cdot\vec{b})\{\exp(i\chi(\vec{b})\cos Y(\vec{b}))-1\} \tag{1}$$

where q is the total momentum transfer, b the impact parameter, and k the relative wave number in the overall center of mass frame. In eq. (1), $\chi(b)$ is the 1st optical model elastic scattering phase function and $Y(b)$ the 2nd-order optical model phase function. The 1st-order phase function can be written as a direct interaction and Pauli exchange terms as [21]

$$\chi(\vec{b}) = \chi_{dir}(\vec{b})-\chi_{ex}(\vec{b}) \tag{2}$$

The direct term is written in terms of the projectile and target ground-state single particle form factors, $F(q)$ and $G(q)$, respectively, and nucleon-nucleon scattering amplitude, $f_{NN}(q)$ as

$$\chi_{dir}(\vec{b}) = \frac{A_PA_T}{2\pi k_{NN}}\int d^2q e^{-i\vec{q}\cdot\vec{b}}F(\vec{q})G(-\vec{q})f_{NN}(\vec{q}) \tag{3}$$

And the Pauli exchange term as

$$\chi_{ex}(\vec{b}) = \frac{A_PA_T}{2\pi k_{NN}}\int d^2q e^{-i\vec{q}\cdot\vec{b}}F(\vec{q})G(-\vec{q})\int d\vec{q}' e^{i\vec{q}'\cdot\vec{b}}f_{NN}(\vec{q}+\vec{q}')C(\vec{q}') \tag{4}$$

The Pauli exclusion correlation factor is represented by [21]

$$C(\vec{q}) = \frac{\pi}{4d}e^{-q^2/4d^2} \tag{5}$$

with $d = 1.85\ \mathrm{fm}^{-1}$.

2.1. 2nd-order optical function terms

The 2nd-order optical phase function considers the scattering of nucleons with two-body correlations in the projectile and target nuclei ground-state and assuming diagonal scattering between excited states is similar to that of the ground-state. The 2nd-order optical phase function is found as [10–12],

$$Y^2(\vec{b}) = \frac{A_PA_T}{(2\pi k_{NN})^2}\int d\vec{q}\,d\vec{q}' e^{i(\vec{q}+\vec{q}')\cdot\vec{b}}\cdot f_{NN}(-\vec{q}')f_{NN}(\vec{q})\sum_{n=1}^{5} y_n(\vec{q},\vec{q}') \tag{6}$$

where

$$y_1(\vec{q},\vec{q}') = -A_PA_TF(\vec{q})F(\vec{q}')G(-\vec{q})G(-\vec{q}') \tag{7}$$

$$y_2(\vec{q},\vec{q}') = (A_P-1)(A_T-1)F_2(\vec{q},\vec{q}')G_2(-\vec{q},-\vec{q}') \tag{8}$$

$$y_3(\vec{q},\vec{q}') = (A_T-1)F(\vec{q}+\vec{q}')G_2(-\vec{q},-\vec{q}') \tag{9}$$

$$y_4(\vec{q},\vec{q}') = (A_P-1)F_2(\vec{q},\vec{q}')G(-\vec{q}-\vec{q}') \tag{10}$$

$$y_5(\vec{q},\vec{q}') = F(\vec{q}+\vec{q}')G(-\vec{q}-\vec{q}') \tag{11}$$

where $F_2(q,q')$ and $G_2(q,q')$ are the two-body form factors for the projectile or target ground-states, respectively.

The terms appearing in $Y^2(\vec{b})$ are evaluated as follows. The first term, $i = 1$, is readily seen to simplify to $-\chi^2(b)$. The last term, $i = 5$, is much smaller than the others for $A_PA_T >> 1$, and can be ignored for reaction pairs considered here. The terms $i = 2, 3, 4$ represent the contributions from two-body nucleon correlations, which require a model for the two-body ground-state density or two-body form factor. Of note is that using form-factors, a 6-dimensional numerical integration occurs or using two-body densities, a 12-dimensional integration occurs. This is compared to the 1st-order terms that readily reduce to a 1-dimensional numerical integration using form-factors, thus suggesting a simplification scheme is needed to implement the 2nd-order calculations.

Introducing the two-particle ground state density:

$$F_2(\vec{q},\vec{q}') = \int d\vec{r}\,d\vec{r}' e^{i\vec{q}\cdot\vec{r}}e^{i\vec{q}'\cdot\vec{r}'}\rho_2(\vec{r},\vec{r}') \tag{13}$$

Defined by

$$\rho(\vec{r},\vec{r}') = \int |\psi(\vec{r},\vec{r}',r_i,\ldots r_{Ap})|^2 d\vec{r}_i\cdots d\vec{r}_{Ap} \tag{14}$$

The usual ansatz used to model the two-particle density as the product of the single particle density for the two nucleon folded with a correlation function [22,23] as,

$$\rho(\vec{r},\vec{r}') \approx \rho(\vec{r})\rho(\vec{r}')[1-n(\vec{r}-\vec{r}')] \tag{15}$$

The correlation function obeys

$$A\int d\vec{r}\,d\vec{r}'\rho(\vec{r})\rho(\vec{r}')[n(\vec{r},\vec{r}')-1] = -1 \tag{16}$$

Which leads to the constraint [23],

$$\int dr[n(\vec{r})-1] \approx -\frac{1}{A\int d\vec{r}\rho^2(\vec{r})} \tag{17}$$

We used a Gaussian function for $n(\vec{r}-\vec{r}')$ and Gaussian single particle densities, and compared two models of the correlation length parameter. Center-of-mass corrections are introduced following the description of Franco and Nutt [23]. The first model is in terms of the Fermi momentum where the correlation length is $p_F^2/5$, and the second model is the approach suggested by Franco and Nutt, 1978. The Fermi momentum (in units of MeV/c) is parameterized as a function of mass number as [24]:

$$p_F = 197.1 + 70.56(1-e^{-0.0355A})\ \text{for}\ A > 10 \tag{18a}$$

$$p_F = 160.9 + 9(A-5)\ \text{for}\ A \leqslant 10 \tag{18b}$$

Comparisons of the two models were similar. The Fermi momentum model provided a smaller correlation length and was used in the absorption model prediction discussed below based on quality of predictions.

2.2. Total and absorption cross sections

The total scattering cross section is the sum of the absorptive and elastic parts:

$$\sigma_{OT} = \sigma_{ABS} + \sigma_{EL} \tag{19}$$

The optical theorem relates the imaginary part of the elastic scattering amplitude to the total cross section by