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Cherenkov and parametric (quasi-Cherenkov) radiation produced by a relativistic charged particle moving through a crystal built from metallic wires

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ABSTRACT

Parametric radiation in THz frequency range, produced by relativistic charged particles moving through a crystal built from metallic wires is considered. Effective polarizabilities of the crystal are found. The radiation intensity is shown to be maximal at $\lambda \sim 2\pi R$, where R is the wire radius and λ is the radiation wavelength. The peak power of Cherenkov and parametric radiation produced by electron bunches in such a crystal at $\lambda \sim 2\pi R$ are demonstrated to be high enough to enable experimental observation and practical applications.

1. Introduction

Emission of photons by relativistic charged particles moving in natural crystals or artificial spatially periodic structures (photonic crystals, metamaterials) was the subject of numerous studies in recent years. It was found that spatially periodic structure of a crystal could facilitate supplementary mechanisms of radiation from a uniformly moving particle to occur in addition to well-known Cherenkov and transition radiation. They are: diffraction radiation [1–3], parametric X-ray radiation (sometimes referred to as "quasi-Cherenkov" radiation) [4,5] and the Smith-Purcell effect, which originates from a particle moving along the crystal surface [6].

In [7–11] radiation produced by charged particles moving in a crystal built from parallel metallic wires (wire media) for wavelengths $\lambda = 2\pi/k$ much greater than both the wire radius R and the crystal period d ($kR \ll 1, kd \ll 1$) was considered. Therein the crystal was presented as a uniform medium characterized by certain permittivity and permeability tensors.

Radiation in crystals built from parallel metallic wires for case, when diffraction effects are important, was studied in [12,13]. According to [13], when the wavelength is comparable with the wire radius ($kR \sim 1$) a noticeable increase in the intensity of parametric radiation is expected.

The analysis [13] was based on extrapolation of the results obtained for the diffraction theory valid at $kR \ll 1$ to the range $kR \sim 1$. However, the manner of the interaction of electromagnetic waves with the wire media changes substantially when parameter kR increases. Particularly, in contrast to case $kR \ll 1$, the scattering by a single wire is anisotropic even for the electromagnetic wave, which polarization is parallel to the wire axis, when $kR \sim 1$. The equations describing dynamical diffraction in 2D crystals, which are valid for the case of anisotropic scattering by a single constituent element of the crystal (e.g. wire), were first obtained in [14].

In the present paper refraction and diffraction of waves in crystals built from metallic wires in the case when $kR \sim 1$ are analyzed on the basis of theory [14]. Spontaneous radiation from charged particles moving in such crystals is studied for THz frequency range. In agreement with [13], radiation intensity is shown increasing with increase of wire radius, achieving its maximum in the range $kR \sim 1$. Several examples differing in crystal thickness, radius of wires and energy of particles are considered. Maximum values of intensity for parametric and Cherenkov radiation are calculated.

2. Propagation of waves in a crystal built from parallel metallic wires

Let a plane electromagnetic wave $\mathbf{E}_0 = \mathbf{e}_0 e^{i\mathbf{kr}}$ be scattered by an infinite cylinder of radius R. It is assumed that the axis of the cylinder coincides with the x-axis of the Cartesian rectangular

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coordinates and $\mathbf{k} = (k_x, 0, k_z)$. We shall also introduce a polar coordinate system (ρ, φ) in the (z, y)-plane using the relations $z = \rho\cos\varphi$ and $y = \rho\sin\varphi$.

It is necessary to consider two possible polarizations of the incident wave:

- transverse electric (TE) polarization, when the vector $\mathbf{E}_0$ of the electric field strength is perpendicular to the axis of the cylinder ($E_{0x} = 0$). Hereinafter the quantities referring to this polarization will bear the index "⊥";
- transverse magnetic (TM) polarization, when the vector $\mathbf{H}_0$ of the magnetic-field strength is perpendicular to the axis of the cylinder ($H_{0x} = 0$). Hereinafter the quantities referring to this polarization will bear the index "‖", since in this case vector $\mathbf{E}$ has a nonzero component which is parallel to the cylinder axis x.

To solve scattering problems, E_x and H_x are usually presented as a series in terms of cylindrical functions [15,16], while other unknown elements of the fields are expressed in terms of E_x and H_x using Maxwell's equations. Let us suppose that $|k_x| \ll k_\rho \equiv \sqrt{k_y^2 + k_z^2}$. Then, according to [14], the wave scattered by the cylinder at $k\rho \gg 1$ can be written in the form:

$$\Psi = \left\{ \begin{array}{c} E_x \\ H_x \end{array} \right\} = e^{i\mathbf{k}_\rho \boldsymbol{\rho}} + \left\{ \begin{array}{c} A^{\parallel}(\varphi) \\ A^{\perp}(\varphi) \end{array} \right\} \times \int_{-\infty}^{\infty} \frac{e^{ik\sqrt{\rho^2+x^2}}}{\sqrt{\rho^2+x^2}} dx, \quad (1)$$

where $A^{\parallel(\perp)}(\varphi) = \sum_{n=0}^{\infty} A_n^{\parallel(\perp)} \cos(n\varphi)$ is the amplitude of scattering of the electromagnetic wave by a cylinder at an angle φ. The coefficients A_n depend on parameter $k_\rho R$ and on permittivity and permeability of wire material; complete expressions for A_n can be found in [15–17].

Further analysis is confined to the case $0 < k_\rho R \lesssim 1$. In this range, we can take into account only the series terms with $n = 0, 1$, because all other terms are small [17]. As a result, the expression for a wave scattered by a wire with the coordinates $\rho_0 = (y_0, z_0)$ can be written in the form:

$$\Psi = e^{i\mathbf{k}_\rho} + i\pi A_0 H_0(k_\rho|\rho - \rho_0|) - \pi A_1 H_1(k_\rho|\rho - \rho_0|) \times \cos(\mathbf{k}_\rho, \rho - \rho_0), \quad (2)$$

where $H_{0,1}$ are the Hankel cylindrical functions of the first kind. It should be noted, that in case when $k_\rho R \ll 1$ for TM wave $|A_1| \ll |A_0|$ and, thus, scattering by a single wire is isotropic (amplitude A does not depend on φ). In case under consideration, on the contrary, scattering by a single wire is anisotropic.

Let us consider the wave propagation in a crystal built from parallel metallic wires. We assume that wire radius R is much less than the crystal period. Analysis of refraction and diffraction in crystal can be carried out, when its effective dielectric permittivity is known. Due to permittivity $\varepsilon(\mathbf{r}, \omega)$ of a crystal built from parallel wires being a 2D-periodic function it can be expanded into Fourier series as follows:

$$\varepsilon(\mathbf{r}, \omega) = 1 + \sum_{\tau} g_\tau(\omega) e^{-i\tau\mathbf{r}}, \quad (3)$$

where τ are the reciprocal lattice vectors of the crystal. To describe refraction in the crystal it is sufficient to know its effective electric susceptibility g_0; diffraction in the crystal can be described when effective polarizabilities of the crystal g_τ are known.

In a general case, diffraction of a plane wave by a one-dimensional grating formed by periodically placed parallel wires, gives rise to several plane waves diverging from the grating, whose amplitudes are expressed in terms of the scattering amplitudes $A_{0,1}$ [14,17]. Consideration of the plane-wave scattering by the crystal, consisting of a great number of one-dimensional gratings (crystal planes), provides us with the dispersion equation relating the frequency ω with values of the wave vector in crystal $\mathbf{q}$ [17]. Fig. 1 gives an example of calculated g_0 and g_τ using dispersion equation [17]. We consider a crystal with a square lattice with period d and it is assumed that Bragg conditions in the crystal are fulfilled for two waves (two-wave diffraction). The left graph in Fig. 1 shows the dispersion curves for TM wave at $R/d = 5 \cdot 10^{-3}$. The bold curve represents the solution of the dispersion equation for vacuum ($k^2 = k_z^2 + k_y^2, k_z \equiv q_z$). The inset image shows the range of high frequencies ($kd \gg 1$) and indicates two roots corresponding to two close solutions of the dispersion equations. The right graph in Fig. 1 shows the corresponding absolute values of g_0 and g_τ calculated for the selected geometry with parameter $k_\rho R$ varied in the range $0 < k_\rho R < 1.2$. For TE wave, $|g_0^\perp|$ and $|g_\tau^\perp|$ increase with $k_\rho R$ increasing and attain their maxima in the vicinity of $k_\rho R \sim 1$. For TM wave, the absolute values $|g_0^\parallel|$ and $|g_\tau^\parallel|$ increase monotonically. Let us note here that for TM wave, the values of g_0 and g_τ are negative, while for TE wave they are positive (i.e., the refractive index for TE wave is greater than unity). In the considered case, the maximum values of $g_0^\perp$ and $g_\tau^\perp$ even exceed the corresponding values $|g_0^\parallel|$ and $|g_\tau^\parallel|$, though in general case the relation between these values can change. However, it is important that in our calculations, the behavior of effective polarizabilities remains almost the same: for TM wave $|g_0|$ and $|g_\tau|$ increase monotonically as $k_\rho R$ is increased from 0 to 1, while for a TE wave both $|g_0|$ and $|g_\tau|$ have maxima at $k_\rho R \sim 1$.

3. Radiation in crystals built from wires

Let a relativistic particle with charge eQ move at a constant velocity $\mathbf{v}$ in a crystal built from parallel metallic wires, as shown in Fig. 2. The crystal thickness L is assumed to be much smaller than its transverse dimensions.

Since the refractive index for a TE wave is greater than unity, Cherenkov radiation is emitted in the crystal [18,19]. Transition radiation is also emitted when a charged particle crosses the "crystal-vacuum" boundary. To calculate spectral-angular distribution of Cherenkov and transition radiations the following expression, which is valid for $|g_0| \ll 1$, can be used [20,21]:

$$\frac{d^2N^s}{d\omega d\Omega} = \frac{e^2Q^2\omega}{4\pi^2\hbar c^3}(\mathbf{e}_s\mathbf{v})^2 \left| e^{i\frac{\omega L g_0^s}{2c\gamma_0}} \left[\frac{1}{\omega - \mathbf{kv}} - \frac{1}{\omega - \mathbf{qv}} \right] \times \left(e^{i(\omega - \mathbf{qv})\frac{L}{c\gamma_0}} - 1 \right) \right|^2, \quad (4)$$

where the wave vector in the crystal $\mathbf{q} \approx \mathbf{k} + \frac{\omega g_0}{2c\gamma_0}\mathbf{N}$, $\mathbf{N}$ is the normal to the crystal surface, $\gamma_0 = \mathbf{kN}/k$, and $\mathbf{e}_s$ is the polarization unit vector ($s = 1$ for TM-polarization and $s = 2$ for TE-polarization; $\mathbf{e}_2 || [\mathbf{k} \times \mathbf{e}_x]$, $\mathbf{e}_1 || [\mathbf{k} \times \mathbf{e}_2]$). The sum intensity of Cherenkov and transition radiations can be found by numerical integration of (4) with respect to angular coordinates and frequency. By way of example we calculated the sum intensity of Cherenkov and transition radiations in a crystal built from metallic wires using the calculated values of g_0 (Fig. 1). The crystal thickness was $L = 10$ cm, crystal period $d = 2$ mm, the particle (electron) velocity was perpendicular to the crystal surface, $\gamma = 100$, integration was carried in the frequency range $\Delta f/f = 0.05, f = 0.83$ THz. The results are given in Fig. 3 (left). As is seen in the plot, the sum intensity of transition and Cherenkov radiations in a crystal made from metallic wires has a maximum in the vicinity of $k_\rho R \sim 1$ that corresponds to the maximum value of $g_0^\perp$. At $k_\rho R \approx 0.9$, in particular, the total radiation intensity for two polarizations is as high as $6 \cdot 10^{-4}$ photons per one electron.

Further discussion deals with parametric radiation. According to [13], in the Laue case we have the following expression for number of parametric photons in the diffraction direction (Fig. 2(b)):