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Dependence of the probability of close collisions of high-energy charged particles in a bent crystal on the orientation of the crystal

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ABSTRACT

The probability of close collisions (PCC) of high-energy protons in a bent crystal was estimated for different orientations of the crystal with respect to the direction of motion of the particles. This allowed to carry out the comparison of the PCC for three main mechanisms of beam deflection by a bent crystal: planar channeling, volume reflection and stochastic deflection. The comparison showed that for positively charged particles the lowest PCC corresponds to stochastic deflection and the highest PCC corresponds to volume reflection.

1. Introduction

If a high-energy charged particle moves in a crystal near one of the main crystallographic axes or planes, correlations between successive collisions appear. As a result of these correlations the probability of close collisions (PCC) of the particle differs from the PCC in the case of motion in an amorphous medium. Recently in [1] the comparison of the PCC of high-energy positively charged particles in a bent crystal in conditions of planar channeling (PC) and stochastic deflection (SD) was carried out. The comparison shown that in the case of SD the PCC is much smaller than in the case of PC. This result was recently confirmed in an experiment at CERN [2]. In this article we continue the investigation of the PCC of high-energy charged particles in a bent crystal. We analyze the dependence of this probability on the angle between the direction of the crystallographic axis and the initial direction of motion of the beam. This analysis allows us within a single model to compare the PCC for three main mechanisms of beam deflection: planar channeling in a bent crystal (proposed by E.N. Tsyganov in [3,4]), volume reflection (proposed by A.M. Taratin and S.A. Vorobiev in [5]) and stochastic deflection (proposed by A.A. Grinenko and N. F. Shul'ga in [6]).

2. Estimation of the PCC for straight crystal

When a high-energy charged particle moves in a crystal under a small angle with respect to one of the main crystallographic axes, the problem of finding the trajectory of the particle can be simplified by using the model of continuous potential [7]. In this model the potential of atomic strings is averaged over the direction of the crystal axis. This allows to reduce the problem of particle motion in a crystal to the problem of motion in a plane that is orthogonal to the axis. Without loss of generality, we will consider particle motion near the ⟨110⟩ axis of silicon crystal. The orientation of the crystal and the coordinate system that we use in our consideration are shown in Fig. 1. The figure shows that x-axis is orthogonal to the $(1\bar{1}0)$ plane and lies in the bending plane, y-axis is orthogonal to the bending plane (001) and z-axis coincides with the direction of the ⟨110⟩ crystal axis at the point of incidence of the particles on the crystal. So, in our model the potential of the crystal is the sum of continuous potentials of the bent atomic strings, that at the point of incidence of particles on the crystal are parallel to the ⟨110⟩ crystal axis.

Let us start the consideration of the PCC of high-energy positively charged particles in a crystal from the particle motion in an unbent crystal. The comparison between the particle motion parallel to atomic strings (that in the case of a bent crystal corresponds to SD) and PC in this case was done in [1]. The ratio between the PCC in these two regimes of motion was estimated in order of magnitude as $\frac{P_a}{P_{pl}} = \frac{\pi r_T}{a}$, were r_T is the rms atomic thermal vibration amplitude in one direction ($r_T = 0.075$ Å for Si at 293 K), a

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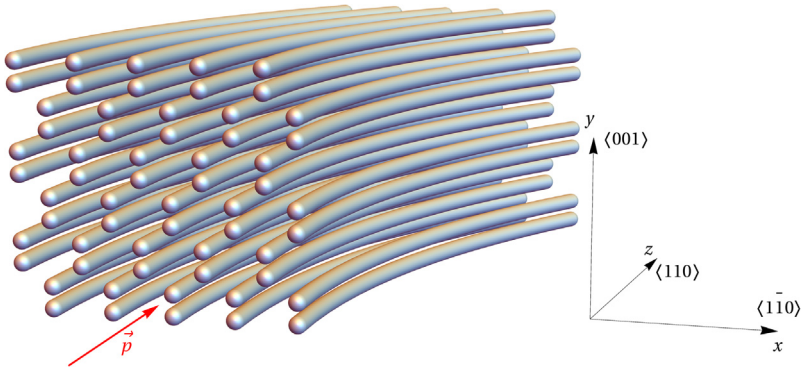

Fig. 1. The orientation of a bent Si crystal with respect to the impinging charged particles.

is the lattice constant ($a = 5.431$ Å for Si crystal). We will now try to compare the PCC in the case of planar channeling and above-barrier motion of a relativistic positively-charged particle in a thin crystal near one of the main crystallographic axes. For analytical consideration of this problem we use a simple approximation of the potential energy of the particle in the field of atomic planes:

$$U(x) = \frac{U_0}{2}\left(1 - \cos\left(2\pi\frac{x}{d_p}\right)\right), \tag{1}$$

where U_0 is the depth of potential well, d_p is the distance between neighboring atomic planes. To find a particle trajectory in the field (1) one must solve the equation of motion

$$\frac{d^2x}{dt^2} = -\frac{c^2}{E}\frac{\partial U(x)}{\partial x}, \tag{2}$$

where E is the energy of the particle. The solution of Eq. (2) can be found as

$$x(t) = \pm\frac{d_p}{\pi}\,\mathrm{am}\left(\frac{\pi c(t+b_2)}{d_p}\sqrt{\frac{U_0}{E}(1+b_1)}, \sqrt{\frac{2}{1+b_1}}\right), \tag{3}$$

where $\mathrm{am}(u,k)$ is the Jacobi amplitude (the inverse function of the elliptic integral of the first kind) [8]. If we assume that in the moment $t = 0$ coordinate x was equal to zero, the parameter b_2 is equal to zero, since $\mathrm{am}(0,k) = 0$. To find b_1, we can find the derivative of $x(t)$ at $t = 0$:

$$v_x(0) = \pm c\sqrt{\frac{U_0}{E}(1+b_1)}, \tag{4}$$

and from (4) we obtain that $b_1 = \frac{E}{U_0}\frac{v_x^2(0)}{c^2} - 1$. Introducing the critical angle of planar channeling [7] as $\theta_c = \sqrt{2U_0/E}$ and the angle $\theta_{x,0} = v_x(0)/c$ we can rewrite (3) as

$$x(t) = \frac{d_p}{\pi}\,\mathrm{am}\left(\frac{\pi t v_x(0)}{d_p}, \frac{\theta_c}{\theta_{x,0}}\right). \tag{5}$$

The character of the dependence of the solution (5) on t is defined by the second argument of the Jacobi amplitude and for different values of $k = \frac{\theta_c}{\theta_{x,0}}$ is shown in Fig. 2. The under-barrier motion (planar channeling) corresponds to $k > 1$, i.e. $\theta_{x,0} < \theta_c$, and the above-barrier motion corresponds to $k < 1$, i.e. $\theta_{x,0} > \theta_c$. If $\theta_{x,0}$ is slightly higher than θ_c (see $k = 0.99$ in Fig. 2) positively charged particles "hang" over atomic planes (n-th plane has coordinate $x_n = \frac{2n+1}{2}d_p, n \in \mathbb{Z}$) (we use the terminology proposed in [10]).

The PCC in a short amorphous target could be written as the product of the collision cross section σ, the atomic density N and the thickness of the target L : $P = \sigma NL$. In a short oriented crystal

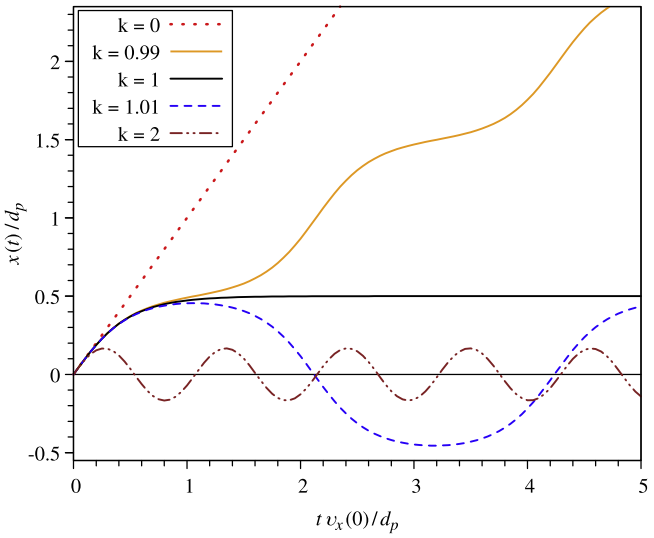

Fig. 2. Positively-charged particle trajectories in the field (1).

the PCC of a high-energy particle can be written in a similar way with a help of the integration over the particle trajectory: $P = \sigma v_z \int_{T_{in}}^{T_{out}} N(x,y,z)dt$, where the atomic density depends from the particle coordinate and we assume that the particle impinges on the crystal at $t = T_{in}$ and withdraws from the crystal at $t = T_{out}$. In the case of motion in the field of atomic planes the atomic density depends only from one coordinate: $N(x,y,z) = N(x)$. If we assume that the atomic density near the atomic plane has a Gaussian distribution with a mean of expectation in $x_n = \frac{2n+1}{2}d_p$ (the coordinate of the n-th atomic plane location), then the PCC can be found as

$$P = \frac{\sigma N v_z d_p}{\sqrt{2\pi r_T^2}}\int_{T_{in}}^{T_{out}}\sum_n \exp\left(-\frac{\left(x(t) - \frac{2n+1}{2}d_p\right)^2}{2r_T^2}\right)dt. \tag{6}$$

If we assume that the crystal is infinite in x direction, the summation over n can be done analytically and thus we obtain

$$P = \sigma N v_z \int_{T_{in}}^{T_{out}} \vartheta_4\left(\pi\frac{x(t)}{d_p}, \exp\left(-\frac{2\pi^2 r_T^2}{d_p}\right)\right)dt, \tag{7}$$

where $\vartheta_4(\zeta, q)$ is the Jacobi Theta Function of the fourth kind [8].

Using the solution of the equation of motion (5) we can find the dependence of the PCC on $\theta_{x,0}$: