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Multicrystal undulator

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ABSTRACT

Radiation of positrons passing through a set of equidistant and mutually oriented crystals is considered. The thickness of each crystal is half of the particle trajectory period at planar channeling in a thick crystal. Passing through the set of half-wave crystals the particle moves on quasi-undulator trajectory. The radiation spectrum of such "multicrystal undulator", consists of discrete harmonics, and the frequency of each harmonic and the number of harmonics in the spectrum depend on the spacing between the crystals, and on the particle energy. Varying the spacing between crystals one can tune the frequency of the first harmonic. A multicrystal undulator should be of particular assistance when there is a need to produce relatively soft radiation by use of a high energy particle beam.

1. Introduction

Channeling radiation of relativistic particles in a single crystal has long been studied as a source of hard electromagnetic radiation. However, new sources of radiation, which combine the advantages of channeling radiation and undulator radiation are of interest today. Especially such that provide possibility for tuning the frequency of radiation. The most common schemes include the crystals periodically bent by means of ultrasonic waves [1,2] or periodic microscopic cuts at a single-crystal plate, which in this case is bent due to the internal stresses [3–5].

Another scheme of crystalline undulator based on a set of thin mutually oriented crystalline plates was proposed in [6]. The thickness of each crystal is half of the particle trajectory period at planar channeling in a thick crystal. Therefore, each crystal of such a "multicrystal undulator" changes the transverse velocity of the particle to reversal one.

It was shown in Ref. [7] that basic properties of radiation emitted in a single half-wave crystal are similar to those of radiation from an arc of a circle [8,9]. However, the first theoretical consideration of radiation generated in a multicrystal undulator [10] has shown that coherent superposition of the radiation fields generated in a series of half-wave plates results in specific properties of the emission spectrum. Further study of radiation emitted in a multicrystal undulator is the objective of this paper. In Section 2 we calculate the trajectory of a positively charged particle in a stack of equidistant and mutually oriented half-wave crystal plates. Each plate reflects the particle in transverse direction keeping the longitudinal velocity constant. As a result the particle moves along a zigzag-shaped path. Section 3 is devoted to investigation of radiation which is emitted by the particle moving this way. In Section 4 we estimate possible input of bremsstrahlung and transition radiation, and we compare the radiation intensity of multicrystal undulator with that emitted in a monocrystal.

2. The particle trajectory

We calculate the properties of a multicrystal undulator using the methods developed in Ref. [10]. The undulator consists of a set of equidistant crystal plates each of half-wave thickness as shown in Fig. 1. The atomic planes which enable channeling of positively charged particles, are orthogonal to the surface of the plates. The particle undergoes half an oscillation in each plate and exits the plate with the reversal of its transverse velocity. Contrary to the paper [10] we consider now the distance between plates d_2 to be much greater than the plates thickness d_1. In this case the point at which the particle enters the next plate is in a large degree random. As a consequence, we can not expect that the sum of the radiation fields emitted in each plate is coherent. Nevertheless, as it will be shown below, the resultant radiation is partly coherent.

We use harmonic approximation for the averaged potential of the atomic planes $U(y) = U_0 y^2/a^2$, where U_0 is the depth of the

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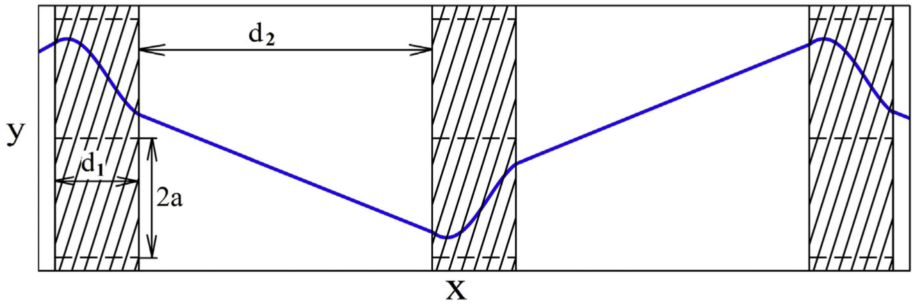

Fig. 1. Diagram of multicrystal undulator. The dotted lines show the atomic planes. The blue solid line is the particle trajectory.

potential valley, $2a$ is the distance between the crystallographic planes. In approximation of small transverse energy the equations of motion have the following solution

$$x(t) = vt, \quad y(t) = \frac{v_{0y}}{\omega_0}\sin\omega_0 t + y_{01}\cos\omega_0 t, \tag{1}$$

where v and v_{0y} are the initial particle velocity and its y-component, α is the incident angle, y_{01} is the initial y-coordinate and ω_0 is the frequency of the particle oscillations $\omega_0^2 = 2eU_0/a^2 m\gamma$, m and e are the mass and the charge of the particle, γ is the relativistic factor. We assume that $\gamma \gg 1$ and $v_{0y} \ll c\gamma^{-1}$.

The thickness of each crystal plate is equal to half of the trajectory period: $d_1 = \pi c/\omega_0$. Hence, the y-coordinate of the particle as it leaves the first crystal plate is $-y_{01}$, and the y-component of the velocity is equal to $-v_{y0}$. Between the first and the second crystal plates the particle moves along a straight line. The law of the particle motion within the second crystal plate has the form

$$y_2(t) = -\frac{v_{0y}}{\omega_0}\sin\omega_0(t-t_2) + y_{02}\cos\omega_0(t-t_2) - b, \tag{2}$$

where $t_2 = (d_1+d_2)/c, |y_{02}| \leqslant a$, and b is a constant which depends on d_2. If $d_2 \gg a$, then $b \approx \alpha d_2$. In order to calculate the properties of radiation we need to know the particle acceleration in each k-th plate

$$\ddot{y}_k(t) = (-1)^{k+1} v_{0y}\omega_0 \sin\omega_0(t-t_k) + y_{0k}\omega_0^2\cos\omega_0(t-t_k), \tag{3}$$

where $t_k = (d_1+d_2)(k-1)/c$. In the case of $d_2 \gg a$ the initial coordinate y_{0k} takes on random values in the interval $[-a, a]$.

3. Radiation

Spectral and angular distribution of the energy radiated by a particle in dipole approximation $\alpha \ll \gamma^{-1}$ is given by formula [11]

$$\frac{d\mathscr{E}}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c(1-\boldsymbol{\beta n})^4}|[\boldsymbol{n}\times[(\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}(\omega')]]|^2, \tag{4}$$

where $\boldsymbol{n}$ is the unit vector pointing to the observer, $\boldsymbol{\beta} = \boldsymbol{v}/c$ is the initial particle velocity, $\omega' = \omega(1-\boldsymbol{\beta n})$ and

$$\dot{\boldsymbol{\beta}}(\omega') = \frac{1}{c}\hat{\boldsymbol{y}}\int_0^T \ddot{y}(t)e^{i\omega' t}dt. \tag{5}$$

Here $T = 2N(d_1+d_2)/c, \hat{\boldsymbol{y}}$ is the unit vector along the y-axis, and N is an integer number of "periods" which the particle traverses before it leaves the channeling mode. Evaluating the last integral by use of Eq. (3) we obtain

$$|\dot{\beta}(\omega')|^2 = 4I_1(\nu)\left|\alpha\frac{\sin\pi N(\nu-1)}{\cos\pi\nu/2} - \frac{1}{c}\omega_0\nu\eta A\right|^2, \tag{6}$$

where

$$I_1(\nu) = \frac{\cos^2\pi\nu\eta/2}{(1-\eta^2\nu^2)^2}, \quad \nu = \frac{\omega}{2\gamma^2\eta\omega_0}(1+\psi^2), \tag{7}$$

$$A = \sum_{k=1}^{N}(-1)^k y_{0k}e^{i\pi\nu k}, \quad \eta = \frac{d_1}{d_1+d_2}. \tag{8}$$

The energy radiated by a beam of particles is an average over particles of the beam. The average value $\langle y_{0k}\rangle$ of y_{0k} vanishes since it varies between $-a$ and a. The mean-square value $\langle y_{0k}^2\rangle$ is equal to $a^2/3$. Hence, the average value of $|\dot{\beta}(\omega')|^2$ is equal to

$$\langle|\dot{\beta}(\omega')|^2\rangle = 4I_1(\nu)\left[\alpha^2 I_2(\nu) + N\frac{(\omega_0\nu\eta a)^2}{3c}\right], \tag{9}$$

where

$$I_2(\nu) = \frac{\sin^2\pi\nu N}{\cos^2(\pi\nu/2)}. \tag{10}$$

The double cross product in Eq. (4) can be reduced to

$$[\boldsymbol{n}\times[(\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}(\omega')]] = [\hat{\boldsymbol{y}}(n_y - 1 + \boldsymbol{\beta n}) + \hat{\boldsymbol{z}}n_y n_z]\dot{\beta}(\omega'). \tag{11}$$

Substituting expression (9) into (11) and (4), we obtain the spectral-angular distribution of radiation of a particle which traverses N "periods"

$$\frac{d\mathscr{E}}{d\Omega d\omega} = \frac{4e^2\gamma^4\alpha_c^2}{\pi^2 c}I_1(\nu)(\rho_\sigma+\rho_\pi)\left[\phi^2 I_2(\nu) + N\frac{\nu^2\eta^2}{3}\right]. \tag{12}$$

Here $\phi = \alpha/\alpha_c$ is the relative angle of incidence, α_c is the critical Lindhard angle $\alpha_c^2 = 2eU_0/m\gamma c^2$, and ρ_σ and ρ_π define the angular distribution of polarization components [12]

$$\rho_\sigma = \frac{(1-\psi^2\cos 2\varphi)^2}{(1+\psi^2)^4}, \quad \rho_\pi = \frac{\psi^4\sin^2 2\varphi}{(1+\psi^2)^4}, \tag{13}$$

where $\psi = \gamma^{-1}\theta$, θ and φ are the polar and azimuthal angles of spherical coordinate system, the polar axes being directed along the x-axis.

The number N is different for different particles due to the beam losses. Hence, we have to average expression (12) over N. Let us denote by p the probability that the particle transmits a half-wave plate. Then the relative beam losses in each plate are equal to $1-p$. The mean value of $\sin^2\pi\nu N$ can be calculated as follows

$$\langle\sin^2\pi\nu N\rangle = (1-p)\sum_{k=1}^{K-1}p^k\sin^2\pi\nu k + p^k\sin^2\pi\nu K,$$

where K is the number of "periods" in multicrystal undulator. Summing up we obtain

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