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Raman scattering of photons by the channeling electrons

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ABSTRACT

The motion of channeling particles in the accompanying coordinate system can be considered as a two-dimensional atom in the case of axial channeling. The transversal motion of the channeling particles is characterized by discrete spectrum. The occupation probability of the transversal motion levels depends on the entrance angle of the charged particles relative to the crystallographic axis. In the scattering of a photon by the "quasi-bound" electron moving in the axial channeling regime would appear the frequencies ω which are a combination of the incident photon frequency ω_0 and the frequency ω_{NM} (ω_{NM} is the transition frequency in transverse quantized motion of the channeling electron: $\omega = \omega_0 \pm \omega_{MN}$, where $\hbar\omega_{MN} = 2\gamma^2\Delta E_{\perp NM}$ for the relativistic electron, $\gamma^2 = E/(mc^2)$ is the Lorentz factor of the channeling electron). In the article are discussed the criteria for choosing an adequate continuous potential of the crystallographic axis and the quantum characteristics of a transversal motion of the channeling electron. The peculiarities of the Raman scattering spectrum of photons by electrons in the axial channeling regime are analyzed and the differential cross section of this process is found.

1. Introduction

A channeling particle moving in the axial channeling regime [1–2] can be represented as "two-dimensional" atom in the accompanying coordinate system (ACS) (i.e. in the coordinate system which moves with the longitudinal velocity of channeling particle: $V_\parallel = p_\parallel/E$, $\beta = V_\parallel/c = p_\parallel/Ec$). From a quantum point of view the channeling particle is characterized with the energy discrete spectrum of the transversal motion [3–4]. The Doppler effect [2–3] affects the system of the energy levels (zones) significantly.

This suggests that a variety of the well-known atomic physics effects may take place for the channeling particles. Particularly, when the channeling particle interacts with a photon the Raman scattering might occur [5–8], which is accompanied with a significant shift in the photon frequency. Previously this effect was investigated in the case of planar channeling positron [9–10]. In this article the Raman scattering of a photon by the axial channeling electron is considered. In a single crystal the monochromatic photon beam can partially be transformed into the photon beam with the frequencies which are harmonics of the fundamental frequency [5]. The Raman scattering of the photon by the channeling electron can be accompanied by the electron transition in quantum states with the greater transversal energy or the smaller one, i.e. the Stocks and anti-Stocks components might be appeared in the photon beam spectrum.

2. The quantum characteristics of the transversal motion of the axial channeling electron

If a charged particle flies into a single crystal at an entrance angle relative to a crystallographic axis less than the Lindhard angle $\theta_L \approx (2U/E)^{1/2}$, then it starts to move in channeling regime [1]. This means that the particle flies quite long distances within the channels formed by the crystallographic axes. The motion across these channels is limited to distances of the order of the lattice constant. From the quantum mechanics point of view the transversal motion will be characterized by a discrete spectrum of energy levels. The basic idea which greatly simplifies the theoretical description of the axial channeling effect is to replace the true potential by the average continuous potential [1,3].

The periodic lattice potential is written in the form:

$$V(\mathbf{R}) = \sum_{\mathbf{g}} V_{\mathbf{g}} \exp(-i\mathbf{g}\mathbf{R}), \tag{2.1}$$

where $\mathbf{g}$ is the reciprocal lattice vector, $\mathbf{R}$ is the radius-vector of the channeling particle: $\mathbf{R} = \mathbf{V}t + \mathbf{r}$, where $\mathbf{r}$ is the radius-vector of the channeling particle (in the accompanying coordinate system moving with longitudinal velocity of the channeling particle $\mathbf{V}$).

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The first term in (2.1) corresponds to the average continuous potential, which ensures the channeling motion [11].

If we assume that the interaction of the charged particle with an isolated lattice atom describes by the screened Coulomb potential

$$V(r) = Z_1 Z_2 e^2 r^{-1} \exp(-r/R_{TF}), \tag{2.2}$$

where Z_1 is the charge of the lattice atom nucleus, Z_2 is charge of the incident particles, $R_{TF}^{-1} = me^2(Z_1^{2/3} + Z_2^{2/3})^{-1/2}/\hbar^2$ is the reciprocal screening radius, then the average potential of isolated row will look

$$\overline{U}(\rho) = 2Z_1 Z_2 e^2 d^{-1} K_0(\rho/R_{TF}), \tag{2.3}$$

where d is the interatomic distance, $K_0(\rho/R_{TF})$ is the Macdonald function.

Expanding the Macdonald function for small values ρ potential (2.3) takes the form

$$\overline{U}(\rho) = Z_1 Z_2 e^2 d^{-1} \ln[(2R_{TF}/(\chi\rho))^2 + 1], \tag{2.4}$$

where $\ln\chi = 0,5772\ldots$ The potential (2.4) is called the standard Lindhard potential.

Let us consider the electron motion in the potential (2.4). The relativistic particle wave function satisfies the Dirac equation

$$[\hbar^2 c^2 \Delta_{\mathbf{r}} + E^2 - m^2 c^4 - 2E\bar{U}(\rho)]\Psi(\mathbf{r}) = 0. \tag{2.5}$$

Despite the simple analytical dependence of the potential (2.4) upon a distance to the crystallographic row, the general solution for the analytical wave functions that satisfy the Eq. (2.5) is quite tedious problem. In this case the simplest and the most effective approximation for the continuous average potential of rows (2.4) is a Coulomb type potential

$$\overline{U}(\rho) = -\lambda Z e^2 R_{TF}/(\rho d)^{-1}, \tag{2.6}$$

where λ is the fitting parameter (Fig. 2.1), Z is the atomic number of the crystal, d is the distance between atoms in the chain.

The eigen wave functions of the transversal motion of the channeling particles are as follows for the potential (2.6) [4]:

$$\Psi_{nl}(\boldsymbol{\rho}) = \rho^{-1/2} \exp[-\beta\rho/a](\rho/a)^{|l|+\frac{1}{2}} \sum_{k=0}^{n} a_k (\rho/a)^k e^{il\varphi}, \tag{2.7}$$

where $n, l = 0, 1, 2, \ldots$ – quantum numbers, $a = \hbar^2 d(\lambda m Z e^2 R_{TF})^{-1}$ is the characteristic length, $m = \gamma m_0$, where m_0 is the electron rest mass, $\rho \equiv \{\rho, \varphi\}$. The factors a_k are discovered as

$$a_{k+1} = 2a_k[\beta(k + |l| + 1/2) - 1](k+1)^{-1}(k + 1 + 2|l|)^{-1},$$
$$\beta = (n + |l| + 1/2)^{-1}. \tag{2.8}$$

The eigenvalues of energy for the potential (2.4) is expressed by the expression

$$E_{\perp N} = -\lambda^2 m Z^2 e^4 R_{TF}^2/[2\hbar^2 d^2 (N + 1/2)^2], \quad N = n + |l|. \tag{2.9}$$

The number of the electron bound motion levels is determines by the relation

$$N' = N_{\max} + 1 = [-m\lambda^2 Z^2 e^4 R_{TF}^2/[2\hbar^2 d^2 \overline{U}(d_r/2)]]^{1/2} + 1/2, \tag{2.10}$$

where N_{max} is the maximum bound state quantum number.

It would be interested to consider the case when there are only two energy levels of the transverse motion. The channeling electrons with energy $E = 1 \div 2$ MeV which interact with the atomic row of the single crystal Si are characterized by the quantum states 1 s, 2 s (Fig. 2.2).

3. Kinematics of the Raman scattering of a photon by the "quasi-bound channeling electron

The Raman scattering of the photon with the frequency ω_1, the wave vector $\boldsymbol{k}_1$ by the channeling electron with momentum $\boldsymbol{p}_1$ and energy E_1 is considered. As a result of the electron-photon interaction the photon with the frequency ω_2, the wave vector $\boldsymbol{k}_2$ is created and the channeling electron turns into the state with momentum $\boldsymbol{p}_2$ and energy E_2.

It would be noted that in the constant field the system energy is conserved (3.2). In a longitudinal direction the average continuous potential, which is responsible for the axial channeling, is constant (the average continuous potential of the atomic row is characterized by the absence of dependence on the longitudinal coordinate [2–3]), so that the longitudinal momentum of the channeling particles is conserved:

$$p_{1z} + \hbar k_{1z} = p_{2z} + \hbar k_{2z}, \quad \omega = kc. \tag{3.1}$$

$$E_1 + \hbar\omega_1 = E_2 + \hbar\omega_2. \tag{3.2}$$

According to the kinematics of the relativistic electron motion

$$p_z = \sqrt{p^2 - p_\perp^2}, \quad p = VE/c, \quad p_\perp = (2EE_\perp/c^2)^{1/2}, \tag{3.3}$$

where $E_\perp$ is the quantum energy of the transversal motion of the channeling electron (2.9) and in the case of the relativistic electron we obtain the equation

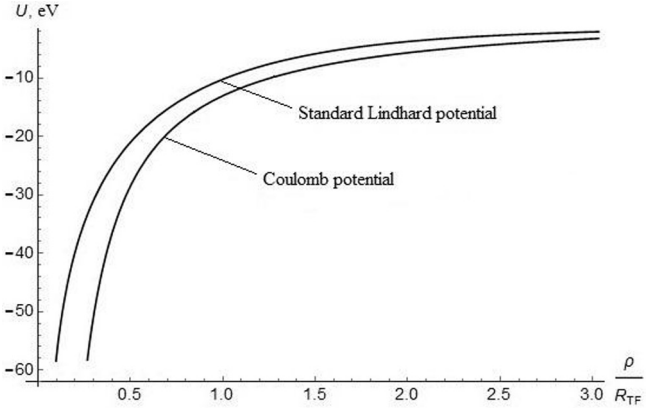

Fig. 2.1. The comparison of the standard Lindhard potential and the Coulomb potential.

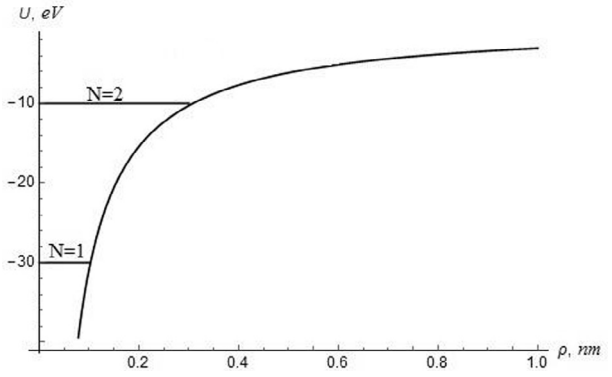

Fig. 2.2. The continuous Coulomb type potential of the relativistic electron interaction with an atomic row ($E = 1$ MeV) and its transversal energy levels.

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