Nuclear Instruments and Methods in Physics Research B 402 (2017) 130–133

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

Effects of correlation in transition radiation of super-short electron bunches

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ARTICLE INFO

Article history:
Received 10 December 2016
Received in revised form 23 February 2017
Accepted 28 February 2017
Available online 27 March 2017

Keywords:
Transition radiation
Super-short bunches
Correlations

ABSTRACT

The effect of correlations between electrons in transition radiation is investigated. The correlation function is obtained with help of the approach similar to the Debye–Hückel theory. The corrections due to correlations are estimated to be near 2–3% for the parameters of future projects SINBAD and FLUTE for bunches with extremely small lengths (~1–10 fs). For the bunches with number of electrons about $\sim 2.5 * 10^{10}$ and more, and short enough that the radiation would be coherent, the corrections due to correlations are predicted to reach 20%.

1. Introduction

Transition radiation (TR) occurs when a charged particle crosses a boundary between two media. It was predicted by Ginzburg and Frank in the last century [1], and the theory of TR was developed in [2,3], etc. It is well-known that in some conditions electrons of bunch can radiate coherently, so it affects the spectral-angular density of TR. In [3] the coherent effects were considered for bunches with comparatively low density. Nowadays, existing technologies make it possible to get electron bunches with extremely high density and small length and size. It is of interest to study how the correlations between the electrons in a bunch can affect the spectral-angular density of TR.

The problem of electrons interaction within the bunches was considered partly in [4] as the space charge effect. The fields of space charge in the electron bunches was calculated, but the intensity of radiation from bunches left non-investigated. In the present paper, the bunch of electrons crossing the interface and radiating coherently is considered. We proceed from the methods set out in [2–4] and construct the theory of TR for correlated bunch, both for the uniform and Gaussian distributions.

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2. Calculations of the correlation part

2.1. The intensity of TR and the coherence factor

Let us consider the spectral-angular density of TR. The expression describing it usually is assumed to consist of two parts: the incoherent part arising due to contribution independently radiating electrons, and the coherent part, caused by the summation of the radiation fields rather than intensities:

$$W_b^{\pm}(\omega,\vartheta) = \frac{1}{2\pi} W^{\pm}(\omega,\vartheta)[N + N(N-1)G_{coh}(\omega,\vartheta)]. \quad (1)$$

Here ω is the frequency of TR, ϑ is the observation angle, $W^{\pm}(\omega,\vartheta)$ is the intensity of TR from one electron ("+" and "–" mean forward and backward parts of radiation, correspondingly), N is the number of electrons in the bunch and $G_{coh}(\omega,\vartheta)$ is the coherence factor. As $G_{coh}(\omega,\vartheta)$ is included in Eq. (1) with the factor $\sim N^2$, it can affect the $W_b^{\pm}(\omega,\vartheta)$ more than the incoherent part.

The coherence factor is determined as follows:

$$G_{coh}(\omega,\vartheta) = \iint d^3r d^3r' f_2(\mathbf{r},\mathbf{r}') \exp\left\{-i\left[\varsigma(\rho_m-\rho_n) + \frac{\omega}{v}(z_m - z_n)\right]\right\}, \quad (2)$$

where $\mathbf{r} = \boldsymbol{\rho} + z\mathbf{n}_z, \mathbf{r}' = \boldsymbol{\rho}' + z'\mathbf{n}_z$ and $f_2(\mathbf{r},\mathbf{r}')$ is the binary electron distribution function normalized to 1. The binary function $f_2(\mathbf{r},\mathbf{r}')$ can be represented as a sum of two parts [5,6]:

$$f_2(\mathbf{r},\mathbf{r}') = f_1(\mathbf{r}) f_1(\mathbf{r}') + f_0(\mathbf{r}-\mathbf{r}'). \quad (3)$$

http://dx.doi.org/10.1016/j.nimb.2017.02.087
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The first part is the function of density of non-interacting particles, whereas the second part corresponds to the coordinate correlations between particles. As a result, one can obtain the following expression for $G_{coh}(\omega,\vartheta)$:

$$G_{coh}(\omega,\vartheta)=G_1(\omega,\vartheta)+G_{cor}(\omega,\vartheta). \quad (4)$$

For the bunch with such a low density that the particles do not repulse each other, the assumption that $G_{coh}(\omega,\vartheta)=G_1(\omega,\vartheta)$ seems to be of common sense. But what if the density is not so low? Below we shall consider it in detail.

With help of Eq. (2) it is possible to obtain expressions for $G_1(\omega,\vartheta)$ both for uniform (Eq. (5)) and Gaussian (Eq. (6)) distributions of particles:

$$G_1^{Uniform}(\omega,\vartheta)=\left|\frac{\sin\left(\frac{\omega a}{2v}\right)}{\frac{\omega a}{2v}}\right|^2\left|\frac{2J_1\left(\frac{\omega}{c}b\sin\vartheta\right)}{\frac{\omega}{c}b\sin\vartheta}\right|^2, \quad (5)$$

$$G_1^{Gauss}(\omega,\vartheta)=\exp\left\{-\frac{\omega^2}{2}\left(\frac{a^2}{4v^2}+\frac{b^2}{c^2}\sin^2\vartheta\right)\right\}. \quad (6)$$

In these equations a and b are the length and transversal size of the bunch, correspondingly, and v is its velocity.

2.2. The correlation part of the coherence factor

Now let us consider only the correlation part of the coherence factor. Eqs. (2) and (3) show that it depends mainly on the correlation part of the density function. It can be obtained with help of the Debye–Hückel theory approach.

Let us consider the bunch as a weakly non-ideal electron gas. In this case the average energy of the Coulomb repulsion is smaller than the average kinetic energy of the electrons:

$$n\ll\left(\frac{T}{e^2}\right)^3. \quad (7)$$

Now it is needed to solve the system of two equations with self-consistent field of electrons [4]:

$$n=n_{e0}\exp\left(-\frac{e\varphi}{T}\right); \quad (8)$$

$$\Delta\varphi=4\pi e\delta(\mathbf{r})+4\pi n_{cor}(\mathbf{r}). \quad (9)$$

Here n_{e0} is the electron density without correlations and n_{cor} is the correlation part of it. Solving the system of Eqs. (8) and (9), one can obtain the correlation part of the density function:

$$f_0(r)=\frac{n_{e0}e}{T}\frac{e^{-\kappa r}}{r}, \quad (10)$$

where

$$\kappa=\frac{1}{r_d}=\sqrt{\frac{4\pi e^2 n_{e0}}{T}}, \quad (11)$$

r_d is the Debye length (Debye radius), which physically is the radius of the screening field.

The correlation part of the coherence factor comes from Eqs. (2) (4):

$$G_{cor}(\omega,\vartheta)=\left|2\pi\iint_{-\infty}^{\infty}dze^{-i\frac{\omega z}{\beta}}\iint_0^{\infty}d\rho f_0(\sqrt{\rho^2+z^2})J_0(\rho\frac{\omega}{c}\sin\vartheta)\rho\right|^2. \quad (12)$$

For the uniform distribution Eq. (12) gives the following expression:

$$G_{cor}^{Uniform}(\omega,\vartheta)=\left(\frac{2e^2}{Tab^2}\right)^2\left|\frac{\sin(\frac{a}{2}\sqrt{\kappa^2+\alpha^2}-i\frac{\omega a}{2v})}{(\sqrt{\kappa^2+\alpha^2})(\sqrt{\kappa^2+\alpha^2}-i\omega/v)}\right|^2 e^{-a\sqrt{\kappa^2+\alpha^2}}, \quad (13)$$

where

$$\alpha=\frac{\omega}{c}\sin\vartheta. \quad (14)$$

From this equation, one can see that there are three conditions, when the contribution of $G_{cor}(\omega,\vartheta)$ to the coherence factor is suppressed:

$$\frac{\pi a}{\beta}\gg\lambda, \quad (15)$$

$$\pi a\sin\vartheta\gg\lambda \quad (16)$$

and

$$a\gg 2r_D \quad (17)$$

The condition in Eq. (16) is similar to the condition for non-coherent electron bunches, so one can assume that in case of non-coherently radiating electrons the correlation effects make very small contribution to the intensity of TR. The condition in Eq. (17) requires that the density of the electrons in the bunch would be high enough.

Note that only when all the conditions in Eqs. (15)(17) are not fulfilled, the effect of correlations can be pronounced.

For Gaussian distribution the final expression for the correlation function is:

$$G_{cor}^{Gauss}(\omega,\vartheta)=\left(\frac{4e^2}{ab^2\pi^{1/2}T}\right)^2\left|Erfc\left(\frac{b}{2r_D}\right)J_0\left(\alpha\frac{b^2}{2r_D}\right)e^{b^2/4r_D^2}\frac{\pi b^2 a/2}{\sqrt{a^2/4+b^2}}\right|^2. \quad (18)$$

There is the increasing exponent function in Eq. (18), but the complementary error function compensates it, so that the whole function does not go to the infinity.

3. Numerical estimates

Let us estimate the contribution of the correlation part to the value of the form-factor. First, we shall get a numerical value for the Debye length for different bunches. The temperature in Eq. (11) is considered to be defined through the average kinetic energy [7]. For different values of the Lorentz-factor $\gamma\sim 10^2\div 10^4$, the values of the temperature can be estimated with help of the following expression from Ref. [7]:

$$\frac{1}{4\pi}\frac{I^2}{2Nk_B}=T, \quad (19)$$

where I is the bunch current, N is the number of particles in the bunch, k_B is the Boltzmann constant. The values obtained are very close to each other for different γ and estimated to be about $T\sim 8.34*10^4$ K. So, we obtain the values of the Debye length for the facilities FLUTE and SINBAD, using the parameters given in [8,9]:

The Debye length at FWHM for the Gaussian distribution was estimated in Table 1. Furthermore, let us plot the dependence of G_{cor} on the parameters of the bunch in the FLUTE experiment. For that we first consider Eqs. (13) and (18). In each of these equations,

Table 1
The Debye length for the experiments SINBAD and FLUTE.

Parameters	SINBAD	FLUTE
Bunch length, a, μm	3	0.3
Debye length (uniform distr.), r_d, nm	2000	200
Debye length (Gaussian distr.), r_d, nm	3765	376.5
Electron energy, MeV	100	42