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Radiation from open ended waveguide with dielectric loading

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ABSTRACT

We analyze radiation produced by a single TM mode incident on the open end of a cylindrical waveguide with uniform dielectric filling. This open-ended waveguide is placed inside concentric vacuum waveguide with a larger radius. Rigorous theory describing excitation of transmitted and reflected modes in each domain is developed. We also perform direct numerical simulation of described structures using Comsol Multiphysics code and show that the results obtained by these approaches for millimeter-sized dielectric waveguide coincide with less-than-percent accuracy. The analytical approach is more efficient for calculation of mode structure at high frequencies (up to Terahertz) and high permittivity. We also consider the situation where generated radiation is extracted into free space through the open end of the outer waveguide. We calculate radiation patterns in the far-field zone using both our algorithm and direct simulations and show that these results are in very good agreement too.

1. Introduction

Terahertz radiation is considered as a promising tool for a number of applications in chemistry, biology and others [1,2]. One prospective scheme to emit THz waves is a passing of a short electron bunch (including the case of a bunch train) through a waveguide structure loaded with dielectric [3,4]. In our previous papers we analyzed this problem in approximate formulation [5]. In the present paper we develop rigorous approach for the case where the open-ended waveguide with dielectric loading is placed inside the regular concentric cylindrical waveguide with a larger radius. This structure is excited by a single mode propagating from within the dielectric waveguide.

It should be noted that the analytical methods for investigation of various waveguide discontinuities have been developed several decades ago [6,7]. However, cylindrical structures which are the most convenient for the accelerator technology were rarely studied, especially this is subject to the presence of dielectric loading. Here we employ the modified residue-calculus technique to the case of cylindrical waveguide with uniform dielectric filling and orthogonal cutoff placed inside infinite vacuum waveguide of a larger radius. Together with analytical approach we perform direct numerical simulation of the described problem using Comsol Multiphysics package.

2. Theory and analytical results

We consider a semi-infinite ideally conducting ($\sigma = \infty$) cylindrical waveguide with radius b filled with a homogeneous dielectric ($\varepsilon > 1$) and enclosed by a concentric infinite cylindrical waveguide with a greater radius $a > b$ (Fig. 1). In theoretical study the lengths $L_d$and $L_v$shown in Fig. 1 tend to infinity, but for numerical simulations (see Section 3) they are finite. We suppose that a single TM_{0l} mode is launched from within the dielectric waveguide in the direction of the open end (cylindrical frame ρ, ϕ, z is used). Omitting the time dependence $\exp(-i\omega t)$, we can write the nonzero components of the incident mode in the form

$$H^{(i)}_{\omega\phi} = J_1(\rho j_{0l}/b) e^{-\kappa^{(1)}_{zl} z}, \quad E^{(i)}_{\omega\rho} = c(i\omega\varepsilon)^{-1} \partial H^{(i)}_{\omega\phi}/\partial z,$$
$$E^{(i)}_{\omega z} = ic(\omega\varepsilon\rho)^{-1} \left(H^{(i)}_{\omega\phi} + \rho\, \partial H^{(i)}_{\omega\phi}/\partial\rho \right). \tag{1}$$

where j_{0l} are the zeros of Bessel function $J_0(x)$, $\kappa^{(1)}_{zl} = \sqrt{j^2_{0l} b^{-2} - k^2_0 \varepsilon}$ ($\mathrm{Re}\kappa^{(1)}_{zl} > 0$), $k_0 = \omega/c$. The reflected (1) and transmitted (2), (3) fields in the corresponding domain can be written in the form

$$H^{(1)}_{\omega\phi} = \sum\nolimits^{\infty}_{m=1} B_m J_0(\rho j_{0m}/b) e^{\kappa^{(1)}_{zm} z}, \tag{2}$$
$$H^{(3)}_{\omega\phi} = \sum\nolimits^{\infty}_{m=1} A_m J_0(\rho j_{0m}/a) e^{-\gamma^{(3)}_{zm} z}, \tag{3}$$
$$H^{(2)}_{\omega\phi} = C_0 \rho^{-1} e^{\gamma^{(2)}_{z0} z} + \sum\nolimits^{\infty}_{m=1} C_m Z_m(\rho\chi_m) e^{\gamma^{(2)}_{zm} z}, \tag{4}$$

where $\gamma^{(3)}_{zm} = \sqrt{j^2_{0m} a^{-2} - k^2_0}$, $\gamma^{(2)}_{z0} = -ik_0$, $\gamma^{(2)}_{zm} = \sqrt{\chi^2_m - k^2_0}$, $\mathrm{Re}\gamma^{(2,3)}_{zm} > 0$,

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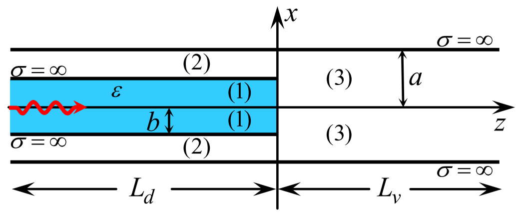

Fig. 1. Geometry of the problem.

$$Z_m(\xi) = J_1(\xi) - N_1(\xi) J_0(a\chi_m) N_0^{-1}(a\chi_m), \tag{5}$$

and $\chi_m$is solution of dispersion relation for domain (2):

$$J_0(b\chi_m)N_0(a\chi_m) - J_0(a\chi_m)N_0(b\chi_m) = 0. \tag{6}$$

In order to find the coefficients $\{A_m\}, \{B_m\}$ and $\{C_m\}$, we perform matching of $H_{\omega\phi}$ and $E_{\omega\rho}$ at $z = 0$ separately for domains $0 < \rho < b$ and $b < \rho < a$, and integrate the obtained relations with eigenfunctions of domains (1) and (2) correspondingly. After series of transformations, we obtain the following infinite systems determining unknown sets of coefficients:

$$\sum_{m=1}^{\infty} \left(\frac{\widetilde{A}_m}{\gamma_{zm}^{(3)} - \gamma_{zp}^{(1)}} + \frac{\widetilde{A}_m R_p}{\gamma_{zm}^{(3)} + \gamma_{zp}^{(1)}} \right) = \frac{-2\delta_{lp} b J_1(j_{0p}) \gamma_{zp}^{(1)} \kappa_{zp}^{(1)}}{\kappa_{zp}^{(1)} + \varepsilon \gamma_{zp}^{(1)}}, \tag{7}$$

$$\sum_{m=1}^{\infty} \left(\frac{\widetilde{A}_m R_p}{\gamma_{zm}^{(3)} - \gamma_{zp}^{(1)}} + \frac{\widetilde{A}_m}{\gamma_{zm}^{(3)} + \gamma_{zp}^{(1)}} \right) = \frac{4\widetilde{B}_p \gamma_{zp}^{(1)} \kappa_{zp}^{(1)}}{\kappa_{zp}^{(1)} + \varepsilon \gamma_{zp}^{(1)}}, \tag{8}$$

$$\sum_{m=1}^{\infty} \frac{\widetilde{A}_m}{\gamma_{zm}^{(3)} - \gamma_{zn}^{(2)}} = 0, \quad \sum_{m=1}^{\infty} \frac{\widetilde{A}_m}{\gamma_{zm}^{(3)} + \gamma_{zn}^{(2)}} = -2\widetilde{C}_n \gamma_{zn}^{(2)}, \tag{9}$$

$p = 1, 2, \ldots$, $n = 0, 1, \ldots$, $\gamma_{zm}^{(1)} = \sqrt{j_{0m}^2 b^{-2} - k_0^2}$, $R_p = \left(\varepsilon\gamma_{zp}^{(1)} - \kappa_{zp}^{(1)}\right)\left(\varepsilon\gamma_{zp}^{(1)} + \kappa_{zp}^{(1)}\right)^{-1}$,

$$\frac{\widetilde{A}_m}{A_m} = J_0\left(\frac{b j_{0m}}{a}\right)\frac{j_{0m}}{a}, \quad \frac{\widetilde{B}_p}{B_p} = \frac{b J_1(j_{0p})}{2},$$
$$\frac{\widetilde{C}_0}{C_0} = \ln\left(\frac{a}{b}\right), \quad \frac{\widetilde{C}_n}{C_n} = \frac{a^2}{2b}\frac{Z_n^2(a\chi_n)}{Z_n(b\chi_n)} - \frac{b}{2} Z_n(b\chi_n). \tag{10}$$

According to the residue-calculus technique [6], in order to simultaneously solve the systems (7)–(9) one should construct the specific function $f(w)$ which satisfies the following conditions:

(i) $f(w)$ is regular in complex plane w excluding first-order poles $w = \gamma_{zp}^{(3)}$;
(ii) $f(w)$ has first-order zeros $w = \gamma_{zn}^{(2)}$;
(iii) $f(w)$ satisfies the relation $f(\gamma_{zp}^{(1)}) + R_p f(-\gamma_{zp}^{(1)}) = 0$ for $p = 1, 2, \ldots$excluding $p = l$;
(iv) for $p = l$ the function $f(w)$ satisfies the relation

$$f(\gamma_{zl}^{(1)}) + R_l f(-\gamma_{zl}^{(1)}) = 2bJ_1(j_{0l})\gamma_{zl}^{(1)}\kappa_{zl}^{(1)}\left(\kappa_{zl}^{(1)} + \varepsilon\gamma_{zl}^{(1)}\right)^{-1}; \tag{11}$$

(v) $f(w) \to_{|w| \to \infty} w^{-(\tau + 1/2)}$, where $\sin(\pi\tau) = (\varepsilon - 1)/(2\varepsilon + 2)$.

Asymptotic (v) follows from Meixner's edge condition for $\rho = b, z \to +0$ [6], this point will be clarified below. As follows from (iii) and (v), the function $f(w)$ possesses first-order zeros $\Gamma_p$shifted with respect to $\gamma_{zp}^{(1)}$:

$$\Gamma_p = \gamma_{zp}^{(1)} + \pi\Delta_p / b, \tag{12}$$

where $\Delta_p$are unknown shifts. The function $f(w)$ can be constructed in the form

$$\frac{f}{G} = P_0\left(w - \gamma_{z0}^{(2)}\right)\prod_{n=1}^{\infty}\left(1 - \frac{w}{\gamma_{zn}^{(2)}}\right)\prod_{s=1}^{\infty(l)}\left(1 - \frac{w}{\Gamma_s}\right)\prod_{m=1}^{\infty}\left(1 - \frac{w}{\gamma_{zm}^{(3)}}\right)^{-1}, \tag{13}$$

$G(w) = \exp\{-w/\pi[b\ln(b/d) + a\ln(d/a)]\}$, $d = a - b$, constant $P_0$is chosen so that (11) is fulfilled, superscript (l) in (13) means that the term with $s = l$ is excluded from the corresponding product. Multiplier $G(w)$ provides algebraic ("physical") behaviour of $f(w)$ at $|w| \to \infty$. Considering integrals over circle $C_\infty$with infinite radius

$$\oint_{C_\infty} \frac{f(w)}{w \pm \gamma_{zn}^{(2)}} dw = \oint_{C_\infty} \left(\frac{f(w)}{w \mp \gamma_{zp}^{(1)}} + \frac{R_p f(w)}{w \pm \gamma_{zp}^{(1)}} \right) dw = 0,$$

calculating them using the residue theorem, and comparing the result with (7)–(9), we obtain

$$\widetilde{A}_p = \operatorname{Res} f(\gamma_{zp}^{(3)}), \quad \widetilde{C}_n = f(-\gamma_{zn}^{(2)})(2\gamma_{zn}^{(2)})^{-1},$$
$$\widetilde{B}_p = -\left(R_p f(\gamma_{zp}^{(1)}) + f(-\gamma_{zp}^{(1)})\right)\left(\varepsilon\gamma_{zp}^{(1)} + \kappa_{zp}^{(1)}\right)\left(4\gamma_{zp}^{(1)}\kappa_{zp}^{(1)}\right)^{-1}. \tag{14}$$

The key problem here is to correctly find shifts Δ_p. To clarify this, one should note that as follows from [6], required (physically correct) behaviour of $E_{\omega z}$near the edge $\rho = b, z \to +0$ is

$$E_{\omega z} \sim z^{-(1/2 - \tau)}, \tag{15}$$

that is dielectric in domain (1) decreases order of singularity compared to the vacuum case ($E_{\omega z} \sim z^{-1/2}$for $\varepsilon = 1$). For $p \gg 1$, one obtains $\gamma_{zp}^{(1)} \approx \kappa_{zp}^{(1)} \approx j_{0p}/b \approx \pi(p - 1/4)/b$, $\gamma_{zp}^{(2)} \approx \chi_p \approx \pi p/d$, $\gamma_{zp}^{(3)} \approx j_{0p}/a \approx \pi(p - 1/4)/a$. Supposing that $A_p \sim p^\eta$for $p \gg 1$, from the decomposition (3) and formulas (1) [6] we obtain:

$$E_{\omega z}^{(3)} \sim \sum_{p=1}^{\infty} p^{\eta + 1/2} \exp(-p\tilde{z}) \sim \tilde{z}^{-(1 + \eta + 1/2)}, \tag{16}$$

where $\tilde{z} = \pi z/a \to +0$. Comparing (16) and (15) we conclude that $\eta = -1 - \tau$, therefore $A_p \sim p^{-(1+\tau)}$, $\widetilde{A}_p \sim A_p\sqrt{p} \sim p^{-(1/2+\tau)}$, which means that the presence of the dielectric in domain (1) enhances the order of decreasing of $A_p$in comparison with the case $\varepsilon = 1$. The asymptotic behaviour of $\gamma_{zp}^{(2)}, \gamma_{zp}^{(3)}$and $\Gamma_p$at $p \to \infty$determines the asymptotic behaviour of $f(w)$ at $|w| \to \infty$and therefore, in accordance with the first relation in (14), decreasing of $A_p$at $p \to \infty$(see [6] for details). One can show that (15) is fulfilled if $f(w)$ behaves as condition (v) dictates. Finally, since the asymptotic of $\gamma_{zp}^{(2)}$and $\gamma_{zp}^{(3)}$are known, the asymptotic of $\Gamma_p$should be: $\Gamma_p \longrightarrow_{p \to \infty} \pi/b(p - 1/4) + \pi\tau/b$, that is $\Delta_p \to \tau$at $p \to \infty$. From (iii), we obtain the following nonlinear system, determining Δ_p:

$$\frac{\pi}{b}\Delta_p = R_p\left(2\gamma_{zp}^{(1)} + \frac{\pi}{b}\Delta_p\right)\prod_{n=1}^{\infty\,(l,p)}\frac{\gamma_{zn}^{(1)} + \gamma_{zp}^{(1)} + \pi\Delta_n/b}{\gamma_{zn}^{(1)} - \gamma_{zp}^{(1)} + \pi\Delta_n/b} \times$$
$$\times \frac{\gamma_{zp}^{(1)} + \gamma_{z0}^{(2)}}{\gamma_{zp}^{(1)} - \gamma_{z0}^{(2)}}\prod_{n=1}^{\infty}\frac{\gamma_{zn}^{(2)} + \gamma_{zp}^{(1)}}{\gamma_{zn}^{(2)} - \gamma_{zp}^{(1)}}\prod_{m=1}^{\infty}\frac{\gamma_{zm}^{(3)} - \gamma_{zp}^{(1)}}{\gamma_{zm}^{(3)} + \gamma_{zp}^{(1)}}\frac{1}{G^2(\gamma_{zp}^{(1)})}, \tag{17}$$

$p = 1, 2, \ldots, p \neq l$. System (17) can be solved numerically using iteration process [6,8] with zero-order approximation $\Delta_p = \tau$.

3. Numerical results

Using aforementioned analytical approach, we calculated coefficients of reflected (B_n) and transmitted (A_p, C_n) modes. For comparison with results of direct numerical simulation, we calculated powers carrying by each mode through the corresponding cross-section:

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