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Radiation from a charge rotating inside a cylindrical grating

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ABSTRACT

We investigate the spectral-angular distribution for the radiation emitted by a point charge moving along a helical trajectory inside a cylindrical grating with conducting strips. Two types of the radiation processes are realized: undulator and Smith–Purcell radiations. Their relative contributions to the total radiation intensity are discussed in various asymptotic regions of the parameters describing the diffraction grating and for large harmonics. The region of the parameters is specified for which the interference effects between the undulator and Smith–Purcell radiations are essential.

1. Introduction

The radiation from a charged particle moving along a helical trajectory has been widely discussed in the literature (see [1] and references therein). This type of electron motion is used in helical undulators for the generation of circularly polarized electromagnetic radiation in a narrow spectral interval at frequencies ranging from radio or millimeter waves to X-rays. The corresponding radiation is the main mechanism to explain the emissions of many objects in radio astronomy. In [2–5], a more general exactly solvable problem with the helical motion inside and outside of a dielectric cylinder was considered. It has been shown that under certain conditions for the permittivity of cylinder and the charge velocity, high narrow peaks are present in the spectral-angular distribution for the number of radiated quanta.

In the present paper we consider a problem where the synchrotron radiation from a charge moving along a helix is superposed with the Smith–Purcell radiation (for reviews see [6,7]). Namely, we investigate the radiation from a charged particle moving inside a cylindrical grating with conducting strips parallel to the cylinder axis. It is shown that the presence of the grating gives rise to new interesting features.

2. Geometry of the problem and the electromagnetic fields

Let a point charge q moves along a helical trajectory of radius r_e inside a coaxial grating consisting of metallic strips situated on a cylindrical surface with radius r_c, $r_c > r_e$. The strips with the width a and with the separation b are parallel to the helix axis. In the cylindrical coordinates (r, φ, z), with the axis z along the grating axis, the strips are located in the angular regions $2\pi m/N \leqslant \varphi \leqslant \varphi_0 + 2\pi m/N$, $m = 0, 1, 2, \ldots, N$, where $N = 2\pi r_c/(a+b)$ is the number of periods in the grating and $\varphi_0 = a/r_c$ is the angular width of the strips. We assume that the system is immersed in a homogeneous medium with dielectric permittivity ε. The radius of the helix and the angular velocity ω_0 are expressed in terms of the external magnetic field $H_{\rm ext}$ and of the charge transversal velocity $v_\perp$ by the formulas $r_e = (m_e c v_\perp \gamma)/(qH_{\rm ext})$, $\omega_0 = v_\perp/r_e$, with $\gamma = E_e/m_e c^2$, where m_e and E_e are the mass and the energy of the charge.

Unlike to the problems with a solid cylinder discussed in [2–5], for the problem under consideration an exact solution is not available. In the discussion below we assume that the radius of the helical trajectory is sufficiently close to the cylindrical grating ($r_c - r_e$ is much smaller than the radiation wavelength). In this case, if the angular coordinate of the charge is in the range $2\pi m/N \leqslant \varphi \leqslant \varphi_0 + 2\pi m/N$ (the charge moves near the strips) its field in the region $r > r_c$ is screened by the strip (this screening is exact in the problem with a conducting cylindrical shell). Consequently, we can approximate the electromagnetic field in the

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region outside the grating by the field of the source with the current density ($v_1 = 0,\ v_2 = v_\perp,\ v_3 = v_\parallel$)

$$j_l = (qv_l/r)\delta(r - r_e)\delta(\varphi - \omega_0 t)\delta(z - v_\parallel t), \quad l = 1,2,3, \tag{1}$$

for $\varphi_0 + 2\pi m/N \leqslant \varphi \leqslant 2\pi(m+1)/N$ (corresponding to the segments of the charge trajectory in the regions between the strips) and $j_l = 0$ for $2\pi m/N \leqslant \varphi \leqslant \varphi_0 + 2\pi m/N$.

Within the framework of the approximation we use, the problem is reduced to the evaluation of the electromagnetic field generated by the current density (1) for a source moving in a homogeneous medium. For the vector potential one has

$$A_l(\mathbf{r},t) = -\int d\mathbf{r}'dt'\sum_{l'=1}^{3} G_{ll'}(\mathbf{r},t,\mathbf{r}',t')\frac{j_{l'}(\mathbf{r}',t')}{2\pi^2 c}, \tag{2}$$

where $G_{ll'}(\mathbf{r},t,\mathbf{r}',t')$ is the electromagnetic field Green function in a homogeneous medium with permittivity ε. For the latter one has the Fourier expansion:

$$G_{ll'}(\mathbf{r},t,\mathbf{r}',t') = \sum_{p=-\infty}^{+\infty}\int_{-\infty}^{+\infty}dk_z\int_{-\infty}^{+\infty}d\omega \times G_{ll'}(p,k_z,\omega,r,r')e^{ip(\varphi-\varphi')+ik_z(z-z')-i\omega(t-t')}. \tag{3}$$

Substituting this expansion into (2) and using (1), the vector potential in the region $r > r_c$ is presented as

$$A_l(\mathbf{r},t) = \frac{q}{\pi c}\sum_{n,m=-\infty}^{+\infty} s_m e^{i(n+Nm)\varphi}\int_{-\infty}^{+\infty}dk_z e^{-i\omega_n(k_z)t} \times e^{ik_z z}\sum_{l'=1}^{3} v_{l'}G_{ll'}(n+Nm,k_z,\omega_n(k_z),r,r_e), \tag{4}$$

with the notations $\omega_n(k_z) = n\omega_0 + k_z v_\parallel$ and

$$s_m = \begin{cases} \frac{1}{\pi m}e^{-imN\varphi_0/2}\sin(mN\varphi_0/2), & m \neq 0, \\ N\varphi_0/2\pi - 1, & m = 0, \end{cases}. \tag{5}$$

The expressions for $G_{ll'}(p,k_z,\omega,r,r')$ are known from the literature (see [8]). We omit the details of the evaluation and present the final expressions for the magnetic and electric fields, with the Fourier expansions

$$F_l = \sum_{n,m=-\infty}^{+\infty}\int_{-\infty}^{+\infty}dk_z e^{i(n+mN)\varphi - i\omega_n(k_z)t + ik_z z}F_{nml}, \tag{6}$$

where $F = H, E$ for the magnetic and electric fields, respectively. The Fourier components of the magnetic field in cylindrical coordinates are given by the expressions

$$H_{nm1} = \frac{qs_m}{2c}\left\{-k_z\frac{v_\perp}{2}\sum_{\alpha=\pm1}J_{n+Nm+\alpha}(\lambda r_e)H_{n+Nm+\alpha}(\lambda r) + v_\parallel\frac{n+Nm}{r}J_{n+Nm}(\lambda r_e)H_{n+Nm}(\lambda r)\right\},$$
$$H_{nm2} = \frac{iqs_m}{2c}\left\{k_z\frac{v_\perp}{2}\sum_{\alpha=\pm1}\alpha J_{n+Nm+\alpha}(\lambda r_e)H_{n+Nm+\alpha}(\lambda r) + v_\parallel\lambda J_{n+Nm}(\lambda r_e)H'_{n+Nm}(\lambda r)\right\},$$
$$H_{nm3} = \frac{iqv_\perp\lambda}{2c}s_m J'_{n+Nm}(\lambda r_e)H_{n+Nm}(\lambda r), \tag{7}$$

where $J_\nu(x)$ is the Bessel function, $H_\nu(x) = H_\nu^{(1)}(x)$ is the Hankel function and $\lambda = \sqrt{\omega_n^2(k_z)\varepsilon/c^2 - k_z^2}$. For the Fourier components of the electric field one gets

$$E_{nm1} = \frac{-iqs_m}{2\varepsilon\omega_n(k_z)}\left\{v_\perp\lambda\frac{n+Nm}{r}J'_{n+Nm}(\lambda r_e)H_{n+Nm}(\lambda r) - \frac{v_\perp}{2}k_z^2\sum_{\alpha=\pm1}\alpha J_{n+Nm+\alpha}(\lambda r_e)H_{n+Nm+\alpha}(\lambda r) - v_\parallel\lambda k_z J_{n+Nm}(\lambda r_e)H'_{n+Nm}(\lambda r)\right\},$$
$$E_{nm2} = \frac{-qs_m}{2\varepsilon\omega_n(k_z)}\left\{v_\parallel k_z\frac{n+Nm}{r}J_{n+Nm}(\lambda r_e)H_{n+Nm}(\lambda r) - \frac{v_\perp}{2}k_z^2\sum_{\alpha=\pm1}J_{n+Nm+\alpha}(\lambda r_e)H_{n+Nm+\alpha}(\lambda r) - v_\perp\lambda^2 J'_{n+Nm}(\lambda r_e)H'_{n+Nm}(\lambda r)\right\},$$
$$E_{nm3} = \frac{qs_m}{2\varepsilon\omega_n(k_z)}\left(\lambda^2 v_\parallel - v_\perp k_z\frac{n+Nm}{r_e}\right)\times J_{n+Nm}(\lambda r_e)H_{n+Nm}(\lambda r). \tag{8}$$

The special case $\varphi_0 = 0$ corresponds to the absence of the diffraction grating. In this case $s_m = -\delta_{m0}$ and from (7) and (8) we obtain the fields for the charge moving in a homogeneous medium.

3. Radiation intensity at large distances

In this section we consider the radiation intensity at large distances from the grating. By taking into account that for $x \gg 1$ for the Hankel function one has $|H_\nu(ix)| \approx \sqrt{2/\pi x}e^{-x}$, from (7) and (8) we see that for the part of the field corresponding to the radiation at large distances one has $\lambda^2 > 0$. For $\lambda^2 < 0$ the Fourier components are exponentially damped for large r and their contribution to the energy flux through a cylindrical surface with radius r tends to zero for $r \to \infty$ (here the details are similar to that we have used in [2,5] for the problem of the radiation from a charge rotating around a dielectric cylinder.

For the mode $n = 0$ we get $\omega_n(k_z) = k_z v_\parallel$ and this mode contributes to the radiation field only under the Cherenkov condition $\beta_\parallel = v_\parallel\sqrt{\varepsilon}/c > 1$. The corresponding radiation propagates along the Cherenkov angle $\theta = \theta_{\rm Ch} = \arccos(1/\beta_\parallel)$ with respect to the helix axis and the radiation intensity is given by the expression

$$I_{n=0} = \frac{q^2}{v_\parallel}\sum_{m=-\infty}^{+\infty}|s_m|^2 \times \int_{\beta_\parallel>1}d\omega\frac{\omega}{\varepsilon}\left[\beta_\perp^2 J'^2_{Nm}(x_\omega) + \left(\left(\frac{Nmv_\perp}{x_\omega v_\parallel}\right)^2 + \beta_\parallel^2 - 1\right)J^2_{Nm}(x_\omega)\right], \tag{9}$$

where $\beta_\perp = v_\perp\sqrt{\varepsilon}/c$ and $x_\omega = \omega\sqrt{\beta_\parallel^2 - 1}r_e/v_\parallel$. This part corresponds to the Cherenkov radiation.

For the radiation on the harmonics $n \neq 0$, the condition $\lambda^2 > 0$ is reduced to the inequality

$$k_z^2(1 - \beta_\parallel^{-2}) + 2k_z n\omega_0/v_\parallel + (n\omega_0/v_\parallel)^2 > 0. \tag{10}$$

The corresponding solution is expressed in terms of the angular variable θ, $0 \leqslant \theta \leqslant \pi$, as

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