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Polarization characteristics of radiation in both 'light' and conventional undulators

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ABSTRACT

As a rule, an intensity spectrum of undulator radiation (UR) is calculated by using the classical approach, even for electron energy higher than 10 GeV. Such a spectrum is determined by an electron trajectory in an undulator while neglecting radiation loss. Using Planck's law, the UR photon spectrum can be calculated from the obtained intensity spectrum, for both linear and nonlinear regimes.

The electron radiation process in a field of strong electromagnetic waves is considered within the quantum electrodynamics framework, using the Compton scattering process or radiation in a 'light' undulator. A comparison was made of the results from using these two approaches, for UR spectra generated by 250-GeV electrons in an undulator with a 11.5-mm period; this comparison shows that they coincide with high accuracy. The characteristics of the collimated UR beam (i.e. spectrum and circular polarization) were simulated while taking into account the discrete process of photon emission along an electron trajectory in both undulator types. Both spectral photon distributions and polarization dependence on photon energy are 'smoothed', in comparison to that expected for a long undulator—the latter of which considers the ILC positron source (ILC Technical Design Report).

1. Introduction

One possible option for a polarized positron source for use with future International Linear Collider (ILC) is a helical undulator, which is needed to produce a beam of circularly polarized photons with the subsequent generation of longitudinally polarized positrons [1].

As for the considered parameters of the polarized positron source, the energy of electrons is about 150–250 GeV, the period of the undulator is $\lambda_u = 11.5$ mm, the undulator strength parameter is $K \sim 1$, and the number of periods is $N_u = 2 \times 10^4$). The energy of the emitted photons, meanwhile, is $\hbar\omega > 10$ MeV. In other studies [2–4], the undulator radiation (UR) characteristics are calculated according to the formulas of classical electrodynamics—which, generally speaking, require justification, as the photons of UR at the indicated energy level are emitted in a discrete manner. In other words, in every act, the energy of electron changes abruptly. The UR process, at the very least, should therefore be described within the framework of quantum theory.

This study describes the UR, based on an analogy with radiation from electrons in a 'light' undulator [5,6]. That is, the sequential simulation of each event corresponding to the emission of the UR photon of radiation is treated as a process of nonlinear Compton scattering.

2. Theory of undulator radiation in the classical approach

To begin with, let us consider classical UR.

For the sake of simplicity, we consider UR from a helical undulator with a period λ_u, an undulator strength parameter K (nonlinearity parameter), and a number of periods N_u. In such an undulator, the electron trajectory is a helix. In a system where an electron is, on average, at rest (R-system), the trajectory of an electron is circular with the radius [7]:

$$R \approx \frac{K\ \lambda_u}{\pi\gamma_0}, \text{ if } \quad K \sim 1, \tag{1}$$

where γ_0 is the Lorentz factor.

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Using the well-known formulas for synchrotron radiation, one can easily obtain formulas for radiation intensity after Lorentz-transformation from the R-system to the laboratory system [7]:

$$\frac{dW}{d\Omega} = \frac{8\alpha\hbar\omega_0 N_u \gamma_0^4}{\left(1+K^2+\gamma_0^2\theta^2\right)^3} K^2 \sum_{n=1}^{\infty} n^2 \left[J_n'^2(nZ) + \left(\frac{\gamma_0\theta}{K} - \frac{1}{Z}\right)^2 J_n^2(nZ) \right], \quad (2)$$

where $\alpha = 1/137$ is the fine structure constant; $\omega_0 = \frac{2\pi c}{\lambda_u}\left(1 - \frac{1+K^2}{2\gamma_0^2}\right)$ is the frequency of the fundamental harmonic; θ is the angle of the outgoing photon, $Z = \frac{2K\gamma_0\theta}{1+K^2+\gamma_0^2\theta}$; $n = 1,2,3,\ldots$ is the harmonic number; and $J_n(x)$ and $J\prime_n(x)$ are the Bessel function of order n and its derivative, respectively.

The formula (2) is obtained in the small-angle approximation of the outgoing photon for the long undulator ($N_u \gg 1$). There is the well-known relationship connecting the frequency of the n-th harmonic and the outgoing angle [7]:

$$\omega^{(n)} = n \frac{2\gamma_0^2}{1 + (\gamma_0\theta)^2 + K^2} \frac{2\pi c}{\lambda_u}. \quad (3)$$

Using this relationship, it is convenient to transform expression (2) into the spectral distribution ($dW/d\Omega \to dW/d\omega$) and then use a dimensionless spectral variable $S^{(n)}$ instead of a frequency $\omega^{(n)}$:

$$S^{(n)} = \frac{\omega^{(n)}}{2\gamma_0^2\omega_0} = \frac{\omega^{(n)}}{2\gamma_0^2 2\pi c/\lambda_u} = \frac{n}{1+(\gamma_0\theta)^2+K^2}, \quad (4)$$
$$0 \leqslant S^{(n)} \leqslant S_{max}^{(n)}, \quad S_{max}^{(n)} = \frac{n}{1+K^2}.$$

The photon UR spectrum for the n-th harmonic can be obtained from the UR intensity spectrum divided by the emitted photon energy $\hbar\omega^{(n)}$:

$$\frac{dN^{(n)}}{dS^{(n)}} = 4\pi\alpha K^2 N_u \left\{ \frac{\left[n - S^{(n)} 2\left(1+K^2\right)\right]^2}{4S^{(n)} K^2 \left[n - S^{(n)}\left(1+K^2\right)\right]} J_n^2(nZ) + J_n'^2(nZ) \right\}. \quad (5)$$

The number of photons emitted at the n-th harmonic can be calculated by integration of the spectral distribution (5):

$$N^{(n)} = \int_0^{S_{max}^{(n)}} \frac{dN^{(n)}}{dS^{(n)}} dS^{(n)}. \quad (6)$$

The total number of photons N_{tot} per electron in the undulator is:

$$N_{tot} \approx \frac{2}{3}\pi\alpha K^2 N_u. \quad (7)$$

For $N_u \sim 10^4$ and for $K \sim 1, N_{tot}$ is approximately equal to 100.

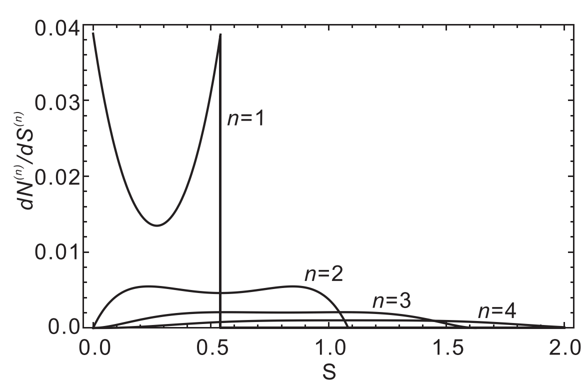

Fig. 1. Spectral distribution for the first four harmonics.

The spectral distributions for different harmonic numbers are presented in Fig. 1; the resulting spectrum is in Fig. 2 (right scale). Both distributions were calculated per an undulator period.

The spectrum (5) depends solely on the parameter K, and has a universal character that allows it to transform into real photon spectra for any values of γ_0, λ_u, and K.

We simulated spectra and polarization of UR for the following parameters [1]:

$$E_0 = \gamma_0 mc^2 = 250\text{GeV}; \ \lambda_u = 1.15\text{cm}; \ N_u = 20000;$$
$$L_u = N_u\lambda_u = 231\text{m}; \ K = 0.92.$$

A collimator with an aperture radius $R_c = 0.7$ mm is placed $L_c = 400$ m from the central point of the undulator [8].

Using the relation (3), we calculated a photon spectrum per unit energy interval, per an undulator period:

$$\frac{dN^{(n)}}{d\hbar\omega^{(n)}} = \frac{\pi\alpha K^2}{2\gamma_0^2\hbar\omega_0} \times \left\{ J_n'^2(nZ) + \left[\frac{\sqrt{2\gamma_0^2\omega_0 n/\omega^{(n)} - 1 - K^2}}{K} - \frac{\gamma_0^2\omega_0 n}{K\omega^{(n)}\sqrt{2\gamma_0^2\omega_0 n/\omega^{(n)} - 1 - K^2}} \right]^2 J_n(nZ) \right\}, \quad (8)$$

where $\omega_0 = 2\pi c/\lambda_u$ (see Fig. 2 (left scale)).

The circular polarization of UR depends on the angle of the outgoing photon and the Stokes parameter ξ_2 characterizes such a polarization component. It can be written as the spectral dependence, using the following relation (4):

$$\xi_2\left(S^{(n)}\right) = \sum_{n=1}^{n_{max}} 2\pi\alpha K N_u \frac{\left[2\left(1+K^2\right)S^{(n)} - n\right]}{\sqrt{S^{(n)}\left[n - S^{(n)}\left(1+K^2\right)\right]}} J_n(nZ) J_n'(nZ) \Big/ \sum_{n=1}^{n_{max}} \frac{dN^{(n)}}{dS^{(n)}}. \quad (9)$$

The dependence of the Stokes parameter ξ_2 on the photon energy for the aforementioned parameters is shown in Fig. 3.

3. The quantum model of the undulator radiation

The process within the classical electrodynamics framework—whereby each treated electron emits ~ 100 photons with energy ~ 10 MeV—needs improvement; this can be had by using the quantum approach.

Other studies [5,6] show the analogy between classical UR and the linear Thomson scattering of the undulator field (approximated by the plane wave) by a relativistic electron. Below, we present further development of such a model for the nonlinear Compton scattering process.

Let us consider the nonlinear Compton backscattering (CBS) process:

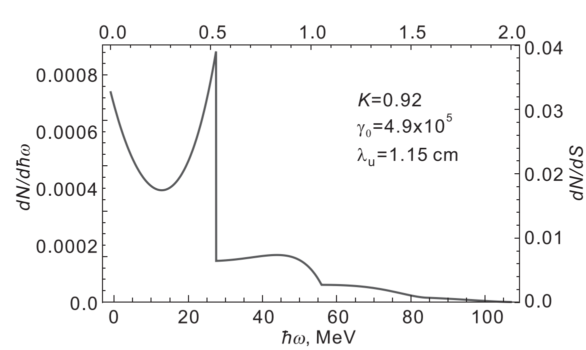

Fig. 2. Resulting spectrum of the summarized harmonics.