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Technical Paper

Prediction of robust stability boundaries for milling operations with extended multi-frequency solution and structured singular values

David Hajdu [a,*], Tamas Insperger [b], Daniel Bachrathy [a], Gabor Stepan [a]

[a] Budapest University of Technology and Economics, Budapest, Hungary
[b] Department of Applied Mechanics, Budapest University of Technology and Economics and MTA-BME Lendület Human Balancing Research Group, Budapest, Hungary

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ABSTRACT

Reliable prediction of machine tool chatter is an essential problem in efficiency-oriented machine tool centers, since it requires the precise characterization of the dynamics of the machine-tool-workpiece system and the cutting force characteristics. Due to imperfect measurements, noise, uncertain and varying operational conditions, the mathematical models provide a deficient representation of the system. This leads to the need for the adaptation of robust stability analysis methods, which guarantee stability against bounded uncertainties and perturbations. In this paper, a frequency-domain approach is presented to calculate the robust stability boundaries of chatter-free machining parameters for milling operations. The idea is based on the concept of the stability radius and structured singular values, which is combined with the extended multi frequency solution. The proposed method is tested in a real case study.

1. Introduction

Industrial manufacturing experiences an increasing competition due to the recent development of powerful machining centers equipped with high-speed spindles and robust slide ways. Optimization of machining processes is an indispensable objective of the efficiency oriented industry. One of the strongest limitation in the industrial utilization of these high-performance machines is the undesired and harmful self-excited vibration, called machine tool chatter, that spoils the surface quality, increases the toolwear and reduces the life-time of the machine components. Reliable prediction of these vibrations is therefore an important task for manufacturing engineers.

The first mathematical models dealing with the self-excited vibrations in machining operations appeared in the work of Tobias [1] and Tlusty [2] in the 1950s and 1960s. After their pioneering research, the so-called regenerative effect became the most commonly accepted explanation for machine tool chatter. During the manufacturing process the vibrating tool leaves a wavy surface behind, which affects the chip thickness and induces variation in the cutting-force one revolution later. From the dynamic system's point of view, chatter is associated with the loss of stability of the stationary (chatter-free) machining process followed by a large amplitude self-excited vibration between the tool and the workpiece.

The stability properties of machining processes are depicted by the so-called stability lobe diagrams, which plot stable (chatter-free) domains in the plane of technological parameters (usually the spindle speed and the depth of cut). These diagrams provide a guide to the machinists to select optimal machining parameters and to avoid undesired vibrations.

There exist several mathematical methods to analyze the stability properties of machining operations and to construct stability lobe diagrams. Some of them apply the measured frequency response functions (FRFs) directly, such as the single-frequency solution or zero-order approximation (ZOA), the multi-frequency solution (MFS) [3,4] or the extended multi-frequency solution (EMFS) [5]. Other time-domain based techniques, such as the semi-discretization method [6,7], the full-discretization method [8], the integration method [9] and their extension by the implicit subspace iteration method [10], the Chebyshev collocation method [11,12] and the spectral element method [13], require fitted modal parameters as input. In spite of the large number of available numerical methods, application of stability lobe diagrams is still not considered to be an essential element of machining. The primary reason for this is that the prediction of chatter-free technological parameters is not reliable enough to convince decision-makers. The input data used for the stability analysis, namely, the dynamics of the

* Corresponding author.
E-mail address: hajdu@mm.bme.hu (D. Hajdu).

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machine tool center and the parameters of the chip removal process model, contain many uncertainties and are loaded with measurement noise. These uncertainties together with model reductions and simplifications lead to an impaired representation of the real dynamical system.

Due to its simplicity, impact test is one of the most commonly used method to characterize the dynamics of the machine tool system and to obtain the frequency response functions. Dynamic measurements by impact tests are, however, affected by several uncertain factors: statistical variations, imperfect calibration coefficients for the hammer and transducer or misalignment between the intended and actual force direction during impact. For a detailed uncertainty analysis of measured FRFs see [14,15].

Despite the need for a reliable method to predict robust stability lobe diagrams, only a few work have been published in this topic. The edge theorem combined with the zero exclusion method is presented in [16,17] and compared to the results obtained by linear matrix inequalities in [18]. A robust chatter prediction method (RCPM) is introduced in [19], which applies a probabilistic approach and considers the parameters as random variables. A different concept based on fuzzy stability analysis is detailed in [20]. The above mentioned techniques require fitted modal parameters (and cutting parameters) as input variables, and the calculation time significantly increases with the number of uncertainties, therefore most of them are limited to systems with few uncertain parameters. In [21], an approximating numerical method is proposed, which provides confidence levels of stability boundaries for higher number of uncertain parameters. Robust stability analysis of turning processes is presented in [22] by means of envelope fitting around the measured FRFs combined with the single-frequency method. This method, however, cannot be applied to time-periodic processes, such as milling operations.

This paper presents a completely frequency-based solution for the robust stability analysis of milling processes, which utilizes directly the uncertainty of the measured frequency response functions and requires no fitted modal parameters. The stability analysis for milling operations is based on the extended multi-frequency solution [4,5], while the robust stability analysis is applied according to the concept of structured singular values (μ-values) [23,24]. The presented algorithm is able to generate robust stability lobe diagrams in reasonable time, which is an advantage in industrial applications.

The structure of the paper is as follows. In Section 2 the dynamical model of milling is introduced. Section 3 gives a detailed description on the stability analysis in frequency domain in a form, which is suitable for the robust stability analysis of the system. The structured singular value calculation is presented in Section 4. The combination of these two concepts gives the new results in Section 5, which also highlights several numerical issues to solve the problem efficiently. The method is tested in a real case study in Section 6. The results are concluded in Section 7.

2. Dynamical model of milling

In this section, cutting force model is presented for conventional helical milling tools with uniform helix angle, which are the most often used type of tools in the industry. Note, however, that the methods introduced in this paper can be extended to tools with nonuniform helix angles [25], variable pitch [26,27], serrated cutter and distributed delay models [28], too.

The helical tool shown in Fig. 1 has N teeth of uniform helix angle β. According to [7], the tool is divided into elementary disks along the axial direction. The relation between the helix angle β, diameter d and the helix pitch l_p is $\tan\beta = d\pi/(Nl_p)$, thus the angular position of the cutting edges along the axial direction reads

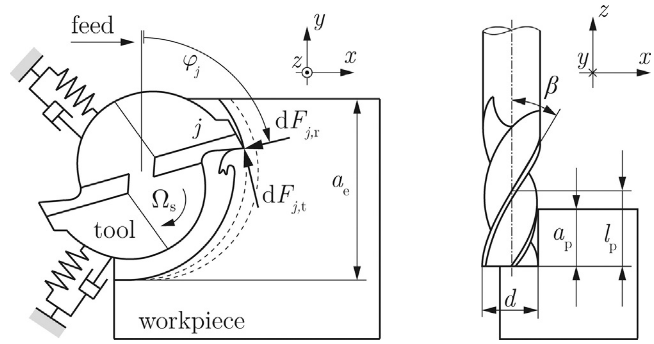

Fig. 1. Dynamical model of milling with rigid workpiece and compliant tool.

$$\varphi_j(t,z) = \frac{2\pi\Omega_s}{60}t + j\frac{2\pi}{N} - z\frac{2\pi}{Nl_p}, \tag{1}$$

where z is the coordinate along the axial immersion and Ω_s is the spindle speed given in rpm. The elementary cutting-force components in tangential and radial directions acting on tooth j at a disk element of width dz are

$$\mathrm{d}F_{j,\mathrm{t}}(t,z) = g_j(t,z)\left(K_{\mathrm{t},e} + K_{\mathrm{t},c}h_j(t,z)\right)\mathrm{d}z, \tag{2}$$

$$\mathrm{d}F_{j,\mathrm{r}}(t,z) = g_j(t,z)\left(K_{\mathrm{r},e} + K_{\mathrm{r},c}h_j(t,z)\right)\mathrm{d}z, \tag{3}$$

where $h_j(t,z)$ is the chip thickness cut by tooth j at axial immersion z, $K_{\mathrm{t},e}$ and $K_{\mathrm{r},e}$ are the tangential and radial edge force coefficients, $K_{\mathrm{t},c}$ and $K_{\mathrm{r},c}$ are cutting force coefficients [29]. The screen function $g_j(t,z)$, which indicates whether the cutting edge is in contact with the material or not, reads

$$g_j(t,z) = \begin{cases} 1, & \mathit{if} \quad \varphi_{en} < (\varphi_j(t,z) \bmod 2\pi) < \varphi_{ex}, \\ 0, & \mathit{otherwise}, \end{cases} \tag{4}$$

where φ_{en} and φ_{ex} are the entry and the exit immersion angles.

The position vector of the center of the tool-tip at time t is denoted by $\mathbf{r}(t) = (x(t)\ y(t))^\top$. The actual chip thickness at tooth j then can be calculated approximately as

$$h_j(t,z) \approx (\mathbf{f}_z + \mathbf{r}(t) - \mathbf{r}(t-\tau))^\top \begin{pmatrix} \sin\varphi_j(t,z) \\ \cos\varphi_j(t,z) \end{pmatrix}, \tag{5}$$

where vector $\mathbf{f}_z = (f_z\ 0)^\top$ describes the feed per tooth in direction x, and the tooth-passing period in case of constant pitch angle is $\tau = 60/(N\Omega_s)$. The resultant cutting force vector $\mathbf{F}(t) = (F_x(t)\ F_y(t))^\top$ can be calculated as

$$\mathbf{F}(t) = -\sum_{j=1}^{N}\int_0^{a_p}\mathbf{T}_j(t,z)\begin{pmatrix} K_{\mathrm{t},e} + K_{\mathrm{t},c}h_j(t,z) \\ K_{\mathrm{r},e} + K_{\mathrm{r},c}h_j(t,z) \end{pmatrix} g_j(t,z)\mathrm{d}z, \tag{6}$$

where the transformation matrix is

$$\mathbf{T}_j(t,z) = \begin{pmatrix} \cos\varphi_j(t,z) & \sin\varphi_j(t,z) \\ -\sin\varphi_j(t,z) & \cos\varphi_j(t,z) \end{pmatrix}. \tag{7}$$

Assuming small perturbation $\varepsilon(t)$ about the periodic motion $\mathbf{r}_p(t) = \mathbf{r}_p(t+\tau)$ of the stationary cutting, i.e. $\mathbf{r}(t) = \mathbf{r}_p(t) + \varepsilon(t)$, the cutting force can be expanded as

$$\mathbf{F}(t) = \mathbf{F}(t)|_{\mathbf{r}_p(t)} + \mathbf{D}_{\mathbf{r}(t)}\mathbf{F}(t)|_{\mathbf{r}_p(t)}(\boldsymbol{\varepsilon}(t) - \boldsymbol{\varepsilon}(t-\tau)), \tag{8}$$

where $\mathbf{D}_{\mathbf{r}(t)}$ is the gradient w.r.t. $\mathbf{r}(t)$ [25]. Derivation for nonlinear cutting force characteristics and generalized milling tools are pre-