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Modeling of Bernoulli production line with the rework loop for transient and steady-state analysis

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ABSTRACT

Production system modeling (PSM) aims to reveal the fundamental principles of production procedures. Most of the research efforts for PSM focus on serial production lines, which is the most fundamental structure in a production system. However, modeling regarding more complex production systems is less developed. Decomposition methods are commonly used to study complex production lines; however, these approaches are only capable of handling steady-state situations. Current closed-form transient modeling cannot be directly applied to production systems with rework loops. In this paper, an analytical method for modeling a production system with a rework loop is presented. Furthermore, a novel 'Self-View' method is proposed for obtaining both transient and steady-state results. This research extends the PSM transient analysis to more complex production lines and overcomes restrictions of machine reliability and buffer capacity in the rework loop. The transient performance measures, i.e., the number of iterations and the settling time, are investigated. Moreover, sensitivity studies of starvation, blockage, production rate, and work-in-process on rework rate are conducted. As a result, the proposed 'Self-View' method in this research shows promising potential for modeling other complex production systems.

1. Introduction

In the past decades, many approaches have been developed to analyze the behavior of production systems [1–6]. Among these approaches, production system modeling (PSM) is an indispensable tool for revealing fundamental production system principles. These principles provide valuable guidelines for production systems designers and open up opportunities for the development of production decision making and operation tools. Thus, much effort has been devoted to PSM.

Existing publications in PSM can be divided into two categories: simulation-based method and analytical method. In simulation-based method, the production systems are often established as discrete event simulation models. The advantage of such models is that they can illustrate the system performance for complex production systems; however, the disadvantage is that the models are not capable of revealing fundamental mathematic production system principles. On the contrary, analytical method can disclose the fundamental relationships between system parameters and performance measures; nevertheless, these relationships are difficult to establish mathematically for complex production systems. Therefore, most analytical PSM methods focus on serial production lines [3,7–9]. Meanwhile, studies concerning more complex production systems modeling are emerging due to their practicality in industry. For example, Helber presents a modeling method for a discrete transfer line with a split of material flows at several stations using an approximate decomposition technique [10]. The basic idea of this method is to transform a long production system into multiple two-machine-one-buffer systems and solve them recursively. In addition, Diamantidis and Papadopoulos propose a Markov process model of a three-machine, one-buffer merge manufacturing system with finite buffer capacity [11].

As one of the complex structures in production systems, rework loops commonly exist in manufacturing industries such as glass, steel, chemical manufacturing, etc. [12], where repair and quality improvement are essential. A typical production system with rework loop consists a main production line and a rework line. In the main production line, the parts are inspected for quality assurance. The defective

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Nomenclature

m_i	The ith machine
p_i	Reliability of machine i
α	Rework rate
$ST_i(t)$	The probability of machine i being starved at time slot t
$T_{i,(j\|k)}(t)$	The probability of buffer i transferring from state k to state j at time slot t
$S_{i,j}$	The probability of buffer i at state j at the steady state
$PR_i(t)$	The production rate of machine i at time slot t
PR_i	The production rate of machine i at the steady state
$WIP_i(t)$	The work-in-process of buffer i at time slot t
$\mathbf{S}_i(t)$	Column vector representing the state of buffer i at time slot t
$\mathbf{X}^f$	The fixed-point of the dynamical system
b_i	The ith buffer
C_i	The capacity of buffer i
$h_i(t)$	The number of parts held by buffer i at time slot t
$BL_i(t)$	The probability of machine i being blocked at time slot t
$S_{i,j}(t)$	The probability of buffer i at state j at time slot t
$T_{i,(j\|k)}$	The probability of buffer i transferring from state k to state j at the steady state
$PR_{SYS}(t)$	The production rate of the production system at time slot t
PR_{SYS}	The production rate of the production system at the steady state
$WIP_{sys}(t)$	The work-in-process of the production system at time slot t
$\mathbf{X}(t)$	The dynamic equations set representing the state of all the buffers at time slot t

parts (if found) are transferred to the rework line for further corrective operations. Once the defective parts are properly processed and satisfy the requirements, they are transferred back to the main production line, where the original manufacturing process resumes. In a production system, rework processes play an important role in eliminating waste and effectively controlling the cost of manufacturing [13].

Given the importance of rework in manufacturing industries, several research studies for modeling the production system with a rework loop have been performed. The author in [14] used a decomposition method to estimate the performance of the production system with a rework loop for the production system only at steady states. Hadjinicola presented a Markovian modeling framework for a serial production system with a rework loop to evaluate the manufacturing cost [15]. However, the buffer capacities in this study are assumed to be infinite, which is unrealistic. The authors in [16] proposed an analytical method on the performance evaluation of a three-machine-one-buffer rework system assuming only one machine exists in the rework line without any buffer. However, their proposed model limited the rework functionality by assuming only one machine exists in the rework line without any buffer. Thus, it was not capable of handling the situation where the defective parts require more than one process before being sent back to the main production line. In summary, current modeling of production systems with rework loops is still far from the real production environment due to adopted impractical assumptions.

In addition to the limitations mentioned above, one important aspect of PSM for production systems with rework loops is inclusion of transients. Transients are the system behaviors before reaching the steady state or transferring from one steady state to another. During the transients, the mean values of the performance measures are not stable and can be quite different from those of during steady state operating conditions [17]. The authors in [18] have shown that a 12% of production loss will occur in a serial production system due to transients for a plant with a work shift of 8 h. The study of transients can reveal the interactions between different parameters within the production system and thus explain production loss due to transients. A few studies are conducted on the modeling of the transient behaviors in production systems. Meerkov and Zhang [18] studied the transient performance for a serial Bernoulli production system with two machines and one buffer. The model proposed in [19] requires that each buffer must have a capacity of one and each machine is 100% reliable. More recently, Wang and Li [3] proposed an analytical model for m-machine-n-buffer serial Bernoulli production system with finite buffer capacity to analyze both steady-state and transient conditions.

Although the number of studies regarding the modeling of either complex system structures or transient behaviors is growing, literature focusing on the integration of these two aspects is lacking. On the one hand, the decomposition method, which is widely used to study complex production lines, is only capable of handling steady-state situations. On the other hand, the current closed-form transient modeling method cannot be directly applied to production system with rework loop. Motivated by the status quo, a 'Self-View' method is proposed in this paper to calculate the exact values of both transient and steady-state performance measures in a Bernoulli production line with rework loops. This method differs from conventional modeling methods by considering each buffer's self-view to calculate its state transition probabilities. In a rework production system, some machines have two downstream buffers and can only transfer the processed part to one buffer at a time. For one of these downstream buffers, as long as it does not receive a part from the upstream machine, it will conclude that the machine is not producing. In conventional modeling approaches, a machine is not producing when it is either starved, blocked or down. However, using the 'Self-View' method, the non-producing machine can have either of the following status: starved, blocked, down, or producing but transferring the part to the other buffer.

The proposed research has four major contributions. First, an analytical approach is presented to calculate both transient and steady-state performance of production systems with rework loops, which will have broad applications in industry. Second, the restrictions in previous modeling of rework production system such as the buffer capacity and the number of machines are overcome. Third, the novel 'Self-View' method proposed in this paper successfully characterizes the dynamics of a production system with rework loop, and shows promising potential to solve other complex structured production systems. Fourth, the model is solved by fixed-point theory, which has