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Riemann's Method in Modelling of Powder Metallurgy Processes

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Abstract

The method of moving coordinates is widely used for determining characteristic nets and, as a result, stress fields in plane strain problems of classical plasticity of rigid plastic material obeying a pressure-independent yield criterion. A great number of boundary value problems related to metal forming processes have been solved by this method. In particular, the method is efficient for constructing the characteristic net in the vicinity of a traction free surface. However, many materials reveal pressure-dependency of the yield criterion, for example materials used in powder metallurgy. The present paper extends the method of moving coordinates to plastically compressible materials that obeys a singular yield criterion and its associated flow rule. This criterion generalizes Tresca's yield criterion. The general problem of determining the state of stress in plane strain deformation is reduced to the equation of telegraphy in characteristic coordinates. This equation can be solved by the method of Riemann. Then, the mapping between the characteristic and Cartesian coordinates is given by simple algebra.

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1. Introduction

The method of moving coordinates (or Mikhlin's variables) is presented in any monograph on plasticity theory, for example [1, 2]. The original method is applicable for plane strain deformation of pressure-independent materials. The Mikhlin variables separately satisfy the equation of telegraphy. This equation can be integrated by the method of Riemann. In particular, the Green's function is the Bessel function of zero order. Solutions found by means of the method of Riemann are quite accurate and can be used for verifying the accuracy of numerical solutions [3]. A great number of solutions of pressure-independent plasticity found by the method of Riemann are provided in [2, 4]. However, many materials reveal pressure-dependency of the yield criterion, for example powder materials [5]. In this case the original method of moving coordinates is not applicable. In the present paper, this method is extended to a class of materials that obeys a piece-wise linear yield criterion that generalizes Tresca's yield criterion on powder materials. It is assumed that material is homogeneous. It is then shown that the generalized Mikhlin's variables satisfy an equation that can be reduced to the equation of telegraphy. Therefore, all methods developed in [1, 2, 4] can be immediately used to solve this equation.

2. System of equations

The method of moving coordinates deals with stress solutions. Therefore, under plane strain deformation the system of equations consists of the equilibrium equations and yield criterion. The yield criterion proposed in [5] as a generalization of Tresca's yield criterion on pressure-dependent materials is

$$\frac{|\sigma_i - \sigma_j|}{2\tau_s} + \frac{|\sigma|}{p_s} = 1, \quad i, j = 1,2,3, \quad i \neq j. \qquad (1)$$

Here σ_1, σ_2 and σ_3 are the principal stresses, σ is the hydrostatic stress, τ_s is the shear yield stress and p_s is the yield stress in hydrostatic compression. In general, both τ_s and p_s depend on the relative density. However, by

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assumption, the material is homogeneous. Therefore, τ_s and p_s are constant. The yield criterion (1) is singular. In particular, several edge and face regimes are possible. It has been shown in [5] that plane stain deformation occurs at one of edge regimes. For powder metallurgy applications it is reasonable to put $\sigma < 0$. With no loss of generality, it is possible to assume that the principal axis corresponding to the principal stress σ_3 is orthogonal to planes of flow and that $\sigma_1 > \sigma_2$. Then, the plane strain yield criterion following from (1) is

$$\frac{\sigma_1 - \sigma_2}{2\tau_s} - \frac{\sigma}{p_s} = 1, \quad \frac{\sigma_3 - \sigma_2}{2\tau_s} - \frac{\sigma}{p_s} = 1. \tag{2}$$

It is evident from this equation that $\sigma_1 = \sigma_3$. This equation and (2) combine to give

$$\sigma_1 (3p_s - 4\tau_s) - \sigma_2 (3p_s + 2\tau_s) = 6\tau_s p_s. \tag{3}$$

This equation and the equilibrium equations constitute the system of stress equations under plane strain conditions. It has been shown in [5] that this system is hyperbolic if $3p_s > 4\tau_s$. In what follows, it is assumed that this inequality is satisfied. The characteristic directions are symmetric relative to the principal stress axes. The angle φ between each of characteristic directions and the axis corresponding to σ_2 is given by

$$\varphi = \arctan \chi, \quad \chi = \frac{3p_s + 2\tau_s}{3p_s - 4\tau_s}. \tag{4}$$

Since, by assumption, both τ_s and p_s are constant, it is seen from (4) that the angle φ is also constant.

3. The plane strain equations referred to the characteristics

There are two distinct characteristic directions at a point. Let the two families of characteristics be labelled by the parameters α and β. The $\alpha-$ and $\beta-$ lines are regarded as a pair of right-handed curvilinear axes of reference. By convention, the line of action of the stress σ_1 falls in the first and third quadrants. The anti-clockwise angular rotation of the principal stress axis corresponding to the stress σ_1 from the x-axis of a Cartesian coordinate system (x, y) is denoted by ψ (Fig. 1). From the geometry of this figure, the $\alpha-$ and $\beta-$ characteristic curves are given by the equations

$$\frac{dy}{dx} = \tan\gamma_\alpha, \quad \frac{dy}{dx} = \tan\gamma_\beta, \tag{5}$$

respectively. Here

$$\gamma_\alpha = \psi + \varphi - \frac{\pi}{2}, \quad \gamma_\beta = \psi - \varphi + \frac{\pi}{2}. \tag{6}$$

The characteristic relations along these lines are [5]

$$\begin{aligned} &\sin 2\chi d(\sigma_1 + \sigma_2) - \frac{6\tau_s}{(3p_s - \tau_s)}(\sigma_1 + \sigma_2 + 2p_s) d\gamma_\alpha = 0, \\ &\sin 2\chi d(\sigma_1 + \sigma_2) + \frac{6\tau_s}{(3p_s - \tau_s)}(\sigma_1 + \sigma_2 + 2p_s) d\gamma_\beta = 0. \end{aligned} \tag{7}$$

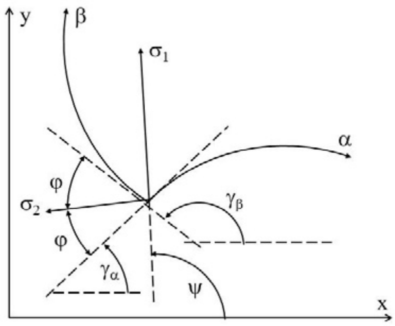

Fig.1. Cartesian and characteristic coordinates.

Since φ is constant, equations (6) and (7) combine to give

$$\begin{aligned} &\sin 2\chi d(\sigma_1 + \sigma_2) - \frac{6\tau_s}{(3p_s - \tau_s)}(\sigma_1 + \sigma_2 + 2p_s) d\psi = 0, \\ &\sin 2\chi d(\sigma_1 + \sigma_2) + \frac{6\tau_s}{(3p_s - \tau_s)}(\sigma_1 + \sigma_2 + 2p_s) d\psi = 0. \end{aligned} \tag{8}$$

The first equation is valid on the $\alpha-$ lines and the second on the $\beta-$ lines. Since χ, p_s and τ_s are constant, the equations in (8) can be immediately integrated to yield

$$k \ln\left(\frac{p}{p_s} + 2\right) - \psi = f_1(\beta), \quad k \ln\left(\frac{p}{p_s} + 2\right) + \psi = f_2(\alpha) \tag{9}$$

where $f_1(\beta)$ and $f_2(\alpha)$ are arbitrary functions of their arguments. Moreover,

$$p = \sigma_1 + \sigma_2, \quad k = \frac{\sin 2\chi (3p_s - \tau_s)}{6\tau_s}. \tag{10}$$

In the case where the two families of characteristics are curved equations (9) are equivalent to