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Shaft-hub press fit subjected to bending couples: Analytical evaluation of the shaft-hub detachment couple

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ABSTRACT

A mathematical modelling of a shaft-hub press-fit subjected to bending couples applied to the shaft extremities is developed, and the value of the bending couple inducing an undesired shaft-hub incipient detachment is analytically determined. The shaft-hub contact is modelled in terms of two elastic Timoshenko beams connected by a distributed elastic spring, whose stiffness is analytically evaluated. Two models of the distributed spring are considered. The first model expresses the combined deformability of both the shaft and the hub cross sections. The second model accounts for the stiffening effect exerted by the shaft portion protruding from the hub on the adjacent shaft part that is in contact with the hub, and, consequently, it assumes only a rigid body motion of the shaft cross section, thus neglecting its deformability.

Based upon this beam-like model, the bending couple producing the incipient detachment between the shaft and the hub is theoretically determined in term of the shaft-hub geometry, of the initial shaft-hub interference, and of the elastic constants. Comparisons with selected Finite Element (FE) forecasts indicate that the first modelling produces an incipient detachment couple that appreciably overrates the FE forecasts, whereas the second modelling lowers the error down to technically acceptable predictions.

1. Introduction

This paper presents a theoretical elastic analysis of a shaft-hub press-fit in the presence of a bending couple C applied to the shaft extremities protruding from the hub, as shown in Fig. 1, and it focuses upon the determination of the bending couple that locally annihilates the initial shaft-hub press-fit, and begins the detachment between the shaft and the hub. The shaft loading by a bending couple in a shaft-hub press-fit is considered in the standards DIN 7190, see [1, Fig. 11.13] and [2, Section 4].

Typical applications of interference fits are to be found in railway rolling stock, in steam turbines, in built-up crankshafts, in drive and powertrain engineering, and in gear and bearing mounting. More generally, assembly by interference may be an effective method of attaching bored members to shafts.

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Nomenclature

a	shaft radius, hub inner radius
$a(r)$	function
b	hub outer radius
$b(\theta)$	function
c	constant
c_i	constant
c_i	coefficients
$f_i(\theta)$	function
$g_i(r)$	function
l	hub half length
l_d	shaft-hub detachment axial length
p	contact pressure
p_{pf}	contact pressure due to the press-fit alone
q_i	contact force
r	radial coordinate
x	coordinate
A_i	area of the cross section
A_0	constant
A_1	constant
B_0	constant
C	bending couple
C_0	constant
D_0	constant
E	Young's modulus
E_0	constant
E_1	constant
F_i	constant
$F_r{}^{(i)}$	body force
$F_\theta{}^{(i)}$	body force
$\boldsymbol{F}^0$	radial and circumferential body force
$\boldsymbol{F}^1$	radial and circumferential body force
G	shear modulus
G_i	constant
H_i	constant
I	diametral interference
I_i	moment of inertia
K	Winkler foundation coefficient
L	hub length
M_i	bending moment
Q	shear force
Q_i	shear force
R	radius of the bore filleted edge
S	variable
U_e	elastic strain energy
V	relative displacement
y	vertical coordinate
z	axial coordinate
α	a/b
γ	parameter
$\gamma_{r\theta}$	shear strain
δ	radial interference
ε	constant
ε_r	radial strain
ε_θ	hoop strain
θ	angular coordinate
κ	Kolosov's constant
λ	parameter