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A novel size independent symplectic analytical singular element for inclined crack terminating at bimaterial interface

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ABSTRACT

Cracks often exist in composite structures, especially at the interface of two different materials. These cracks can significantly affect the load bearing capacity of the structure and lead to premature failure of the structure. In this paper, a novel element for modeling the singular stress state around the inclined interface crack which terminates at the interface is developed. This new singular element is derived based on the explicit form of the high order eigen solution which is, for the first time, determined by using a symplectic approach. The developed singular element is then applied in finite element analysis and the stress intensity factors (SIFs) for a number of crack configurations are derived. It has been concluded that composites with complex geometric configurations of inclined interface cracks can be accurately simulated by the developed method, according to comparison of the results against benchmarks. It has been found that the stiffness matrix of the proposed singular element is independent of the element size and the SIFs of the crack can be solved directly without any post-processing.

1. Introduction

Composite materials are widely used in various engineering sectors due to their optimal properties compared with a single material. However, it is almost inevitable for the composite materials to possess cracks or defects, especially at the interfaces between two joining materials, under fabrication process, environmental degradation, applied loads, etc. The cracks can propagate, accumulate and significantly reduce the load bearing capacity of the structure in terms of fracture failure. Perhaps due to the mathematical complexities in formulating the interface crack, most of the existing studies in literature focused on a special case where the crack is along the interface of the two materials [1–9]. However, a more general and challenging case in which the crack lies in an angle with the interface and terminates at the interface should also be sufficiently addressed. Since the crack can be treated as being inclined from an interface crack, it is named inclined interface crack in this paper.

Stress intensity factors (SIFs) and energy release rates (ERRs) have been widely employed for quantifying the stress singularity around the crack. Cook studied the stress state around the crack terminating at the bimaterial interface, and proposed a fracture criterion for crack initiation [10]. Lin and Mar considered the case in which the crack was perpendicular to the

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Nomenclature

$a^{(i)}$, $i=1, 2, 3,\ldots$	eigen expanding coefficients
$\boldsymbol{a}$	vector of the eigen expanding coefficients
A, B, C, D	coefficients of the general expression of eigenvector
$\boldsymbol{A}$	diagonal matrix of which the elements are composed of eigenvalues
$\boldsymbol{B}$	$\boldsymbol{B}=\boldsymbol{A}(\xi=\ln\rho)$
$\boldsymbol{C}$	matrix of the general expression of eigenvector
$\boldsymbol{d}$	vector of nodal displacements
$\boldsymbol{D}$	vector of the coefficients A, B, C, D
E, ν	Young's modulus and Poisson's ratio
$\boldsymbol{F}$	matrix generated from substituting eigenvector into the boundary condition
G	shearing modulus
$\boldsymbol{G}$	second part of the characteristic equation of eigenvector
$\boldsymbol{H}$	Hamiltonian operator matrix
$\boldsymbol{I}$	identity matrix
$\boldsymbol{J}$	unit symplectic matrix
$k_\alpha, k_\beta, k_\gamma, k_\eta$	$k=p, a, b, c, d$, coefficients in the relationship between the eigenvectors of the region #1 and the region #2
$k'_\alpha, k'_\beta, k'_\gamma, k'_\eta$	$k'=a', b', c', d'$, coefficients in the relationship between the eigenvectors of the region #3 and the region #2
K_I, K_{II}	mode I and mode II stress intensity factors
$\boldsymbol{K}$	stiffness matrix of the SASE
L	characteristic length
M, N	positive integers
$\boldsymbol{p}, \boldsymbol{q}$	vectors of the configuration variables and the dual variables
$\boldsymbol{p}^*, \boldsymbol{q}^*$	vectors of trail functions of the configuration variables and the dual variables
(r, θ)	polar coordinate system
$\boldsymbol{R}$	generalized stiffness matrix of the SASE
$S_r, S_\theta, S_{r\theta}$	generalized stress components
$S_r^*, S_\theta^*, S_{r\theta}^*$	trial functions of thegeneralized stress components
$\boldsymbol{T}_u$	matrix of which the elements are values of eigenvector on the export nodes
u_r, u_θ	displacement components
u_r^*, u_θ^*	trail functions of the displacement components
$\boldsymbol{Z}$	vector of the configuration variables and dual variables
$\boldsymbol{Z}$*	vector of trail functions of the configuration variables and dual variables
α, β	the Dundurs parameters
ε	oscillation index (bimaterial constant)
$\varepsilon_r, \varepsilon_\theta, \varepsilon_{r\theta}$	strain components
$\sigma_r, \sigma_\theta, \sigma_{r\theta}$	stress components
ξ	generalized coordinate
$\theta_1, \theta_2, \theta_3, \theta_4$	$\theta_1=-\pi-\omega$, $\theta_2=-\pi$, $\theta_3=0$, $\theta_4=\pi-\omega$
μ	eigenvalue
$\boldsymbol{\psi}$	eigenvector
$\psi_u, \psi_v, \psi_r, \psi_{r\theta}, \psi_\theta$	variables separated from displacement and stress variables
$\boldsymbol{\Phi}, \boldsymbol{\Theta}$	matrices of which the elements are composed of eigenvectors
$\boldsymbol{\Pi}$	deformation energy of the SASE
ρ	radius of the SASE
η	$\eta=E_2/E_1$, the ratio of the Young's moduli of the two materials
ω	the angle represents crack orientation

material interface. They developed a hybrid element to calculate the SIFs and the ERRs [11]. Chen derived a general expression for the singular stress field around the inclined interface crack, and employed the body force method to calculate the SIFs [12]. In addition, Wang and Per developed a model for solving the SIFs and the T stresses around the crack tip [13]. Lin and Sung [14] and Poonsawat et al. [15] analyzed the stress singularities of the inclined interface crack in anisotropic materials. Furthermore, Wijeyewickrema et al. studied the stress singularities of the crack terminating at the frictional interface of monoclinic bi-material composites [16].

Finite element (FE) analysis is popular in engineering applications; however, if conventional elements were used, extensive mesh refinement around the crack tip would have been employed to ensure the accuracy of simulation [17]. Different