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On the explicit resolution of the mushy zone in the modelling of the continuous casting of alloys

M. Vynnycky[1],*, S. Saleem[2]

Division of Processes, Department of Materials Science and Technology, Royal Institute of Technology, Brinellvägen 23, Stockholm 100 44, Sweden

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ABSTRACT

In the continuous casting of alloys, it is well-known that the mushy zone is decisive for the final properties of the casting. Most numerical models for the process use enthalpy-based methods on fixed grids which determine the extent the mushy zone implicitly. Here, on the other hand, we develop a methodology for explicitly resolving the geometrical extent of the mushy zone; this involves a sharp-interface formulation to solve a dual moving boundary problem to locate the solidus and liquidus isotherms. The results compare favourably with those from enthalpy-based methods, and the advantages of our approach with respect to future multiphysics calculations are discussed.

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1. Introduction

Continuous casting has been developed industrially worldwide since the 1950s as a high throughput method for producing, amongst other things, metal billets, blooms and slabs. In a continuous casting process, such as the strip casting of copper, jets of molten metal enter into the top of a water-cooled mould, where intense cooling causes a solidified metal shell to form [1–3]; subsequently, the metal is withdrawn at a uniform casting speed. The industrial importance of the process has led to interest in modelling, with a view to obtaining an improved understanding of the factors that influence product quality and process productivity.

In general, a model of a continuous casting process, or indeed any other solidification process, must take into account three phases: the molten melt, the solid metal and, in the case of alloys, the mushy zone between them, in which solid and melt coexist [4]. A common numerical technique is to use a fixed grid in tandem with an enthalpy formulation of the governing equations and to employ an auxiliary variable, commonly the local liquid fraction, to track the continuous movement of the phase boundary over the static nodal mesh [5,6]. This method has been used numerous times in literature [7–9], and generally functions very well if a basic description of fluid flow and heat transfer is required.

Often, however, it is necessary to determine quantities that affect the quality of the final solidified alloy, such as the degree of macrosegregation of an alloy's solute elements, the ratio of equiaxed-to-columnar crystals and the thermomechanical stress [10,11]; the first two of these are associated with processes in the mush, whilst the third occurs predominantly in the

* Corresponding author.
E-mail address: michaelv@kth.se (M. Vynnycky).

[1] Present address: Department of Applied Mathematics and Statistics, Institute of Mathematical and Computer Sciences, University of São Paulo at São Carlos, PO Box 668, 13560–970 São Carlos, São Paulo, Brazil.

[2] Present address: Sandvik Heating Technology, Kanthal, Sörkvarnsvägen 3, 734 27 Hallstahammar, Sweden.

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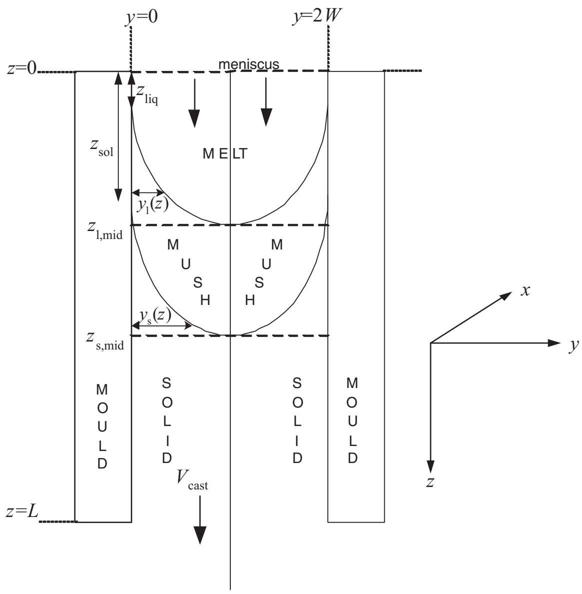

Fig. 1. 2D schematic of the vertical continuous casting of an alloy.

solid, but also in the so-called coherent part of the mush, i.e. where the solid matrix is substantial enough to bear a load. To compute all of these quantities accurately, it would be convenient to be able to resolve the solid-mush and mush-liquid interfaces explicitly. The resolution of one such interface - that between solid and liquid phases - lay behind the computational aspects in a series of recent asymptotically reduced two-dimensional models for the continuous casting of pure copper [12–16]; here, however, we need to extend those ideas to the case of metal alloys where two interfaces will be present, resulting in a dual moving boundary problem. A further consideration is how and where the solidus and liquidus isotherms first appear: where the liquidus isotherm appears, the mushy region will have zero thickness; where the solidus isotherm appears, the solid region will have zero thickness. In the case of pure metal, it has been known for some time that, if there is no superheat, the shell thickness will initially increase as the distance from the start of solidification [17,18]; for the case with superheat, the corresponding result that the shell thickness initially increases as the distance raised to the power 3/2 was determined only quite recently [19]. Consequently, for the case of alloys, the situation can be expected to be entirely different again. Moreover, although we are aware of earlier work where the location of the mush has been explicitly resolved [20,21], this was only for the case of a semi-infinite domain to a constant-temperature cooling condition; in continuous casting, however, neither of these idealizations apply. Therefore, the purpose of this paper is to provide a combined analytical and numerical approach to resolve all of these issues.

The layout is as follows. In Section 2, governing equations are formulated in three different ways; in Section 3, they are nondimensionalized. In Section 4, auxiliary analysis associated with the formation of a new phase, which is explicitly necessary for one of the formulations, is carried out. In Section 5, details of the numerical implementation for the three formulations are provided, and results are given in Section 6. Conclusions are drawn in Section 7.

2. Governing equations

2.1. Full equations (formulation **A**)

We consider a steady state two-dimensional (2D) problem, as shown in Fig. 1, in which an alloy melt at temperature $T_{\rm cast}$, which is greater than or equal to the liquidus temperature, $T_{\rm liq}$, enters a mould region at $z=0$. Mush begins to form at the inner mould surface at $z=z_{\rm liq}$ when the temperature reaches $T_{\rm liq}$, whereas complete solidification occurs at $z=z_{\rm sol}$, where $z_{\rm sol} > z_{\rm liq}$, when the temperature reaches the solidus temperature, $T_{\rm sol}$; on the centreline at $y=W$, mush begins to form at $z=z_{\rm l,mid}$ and complete solidification occurs at $z=z_{\rm s,mid}$. Heat transfer in the solid, liquid and mush regions is expressed through

$$\rho c_{\rm p} V_{\rm cast}\frac{\partial T}{\partial z}=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)-\rho V_{\rm cast}\Delta H_{\rm f}\frac{\partial \chi}{\partial z}, \tag{1}$$

where

$$k=\chi k_{\rm l}+(1-\chi)k_{\rm s}, \tag{2}$$

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