



# Comonotonic global spectral models of gas radiation in non-uniform media based on arbitrary probability measures



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## ABSTRACT

The aim of the present work is to provide a universal theoretical formulation of global methods for radiative heat transfer in non-uniform gaseous media. Starting from the definition of an arbitrary probability measure on the wavenumber axis, it is shown that no gas reference state is required to develop rigorously a full spectrum model, both in uniform and non-uniform media.

This general formulation, which constitutes a novel mathematical modeling of gas radiation, is then applied for: (1) the theoretical justification of new developments introduced recently in the so-called Rank Correlated SLW method in non-uniform media, (2) emphasizing the differences and similarities between the SLW and FSK methods, from the point of view of the way these two techniques treat path non-uniformities.

The theoretical results provided in the present work can also be used to enlighten the concept of “spectral correlation”, widely encountered in gas radiation modeling.

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## 1. Introduction

All existing global methods for the radiative properties of gases, such as ADF [1], SLW [2] or FSK [3], require the specification of some reference thermophysical state of the gas  $\phi_{ref}$ , usually called a state vector and that encompasses all the quantities required to define a given state of the gas, e.g., its temperature, species concentrations and total pressure. The reference state is used to « correlate » spectra at different locations along a non-uniform, i.e., non-homogeneous non-isothermal, radiation path. The reference state  $\phi_{ref}$  needs to be specified in all existing global methods to define reference full spectrum distribution functions (ALBDF in SLW terminology), on which all these techniques [1–3] are founded.

Many approaches [1–3] were proposed during the past decades to choose values for the reference state. However, none of them is fully founded on a systematic rigorous theoretical derivation. Consequently, it is widely recognized that no universal recommendation, other than that based on results obtained on some restricted set of test cases, can be made for general radiative heat transfer calculations in non-uniform gaseous media. We will show here that in fact no reference thermophysical state is actually required to develop a full spectrum gas radiation model. Recourse to sophisticated and mostly theoretically unjustified, but however pragmatic, techniques to define these parameters can thus be avoided.

Indeed, the main objective of the present work is to present a rigorous derivation of a general global method of gas radiation based on an arbitrary probability measure on the wavenumber axis. It does not require specification of the reference

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## Nomenclature

$f$	function of the absorption coefficient
$F$	$k$ -distribution as defined by Eq. (3) or (4)
$Gr$	rank transmutation mapping function, Eq. (5)
$h$	“correlating” function, Eq. (25)
$H$	Heaviside step function
$I$	radiative intensity, Eq. (15)
$I_{b\eta}(T)$	spectral blackbody radiative intensity at temperature $T$ and wavenumber $\eta$
$I_b(T) = \frac{\sigma T^4}{\pi}$	total blackbody radiative intensity, in $W/m^2/sr$
$k$	absorption coefficient, in $cm^{-1}$
$L$	gas path length, in cm
$\mathbb{P}$	probability
$s$	abscissa along a radiation path, in cm
$T$	temperature, in K
<i>Greek symbols</i>	
$\delta$	Dirac Delta function
$\Delta k$	small increment of values of $k$ , in $cm^{-1}$
$\eta$	wavenumber, in $cm^{-1}$
$\kappa_\eta$	spectral absorption coefficient, in $cm^{-1}$
$\phi$	gas thermodynamic state vector $\phi = \{\text{temperature, species concentrations, total pressure}\}$
$\mu$	positive function used to define a measure on the wavenumber axis
$\rho_S$	Spearman’s rank correlation coefficient, Eq. (26)
$w$	weights of the quadratures in Eq. (10)
$\xi$	random variable uniformly distributed over the interval $[0, 1]$

### Subscripts, superscripts

$loc$	local state along a non-uniform path
$P$	Planck mean
$\mu$	relative to the measure $\mu$
$ref$	reference
$i, n$	quadrature index and order respectively
$m, 1, 2$	index representing a particular thermophysical state of the gas
Notation	$\frac{dGr(X=x)}{dX}$ represents the value of the derivative of function $Gr$ with respect to $X$ evaluated at $X=x$ . It is equivalent to $\frac{dGr(X=x)}{dX} = \left[ \frac{dGr(X)}{dX} \right]_{X=x}$ .

thermophysical state. The mathematical evidence is founded on the same tools as introduced in the  $\ell$ -distribution approach [4], i.e., rank transmutation maps and copulas that are two concepts taken from probability theory.

This paper is mostly theoretical. A number of applications of the theoretical development presented here have been presented in [5], where the Rank Correlated SLW model is introduced. Consequently, only limited illustration through examples is included here. The present derivation can serve as a theoretical justification of the « correlated » methods in non-uniform media based on the assumption of Rank Correlated or comonotonic spectra which do not require specification of the reference state. Connections of the present derivations with the Rank Correlated SLW model [5] are emphasized at the end of the document. Solovjov et al. [5] provides extensive applications of the theoretical results developed in the present work and illustrates its relevancy for the building of new and efficient methods of gas radiation.

Finally, this work also has as its objective the careful introduction of the concept of spectral correlation. This problem, together with its engineering treatment, is undoubtedly one of the trickiest concepts in gas radiation modeling. Section 3 is specifically dedicated to this topic.

## 2. Absorption line distribution function

In this section, we consider radiative transfer in a uniform gaseous medium at state  $\phi$  characterized by the spectral absorption coefficient  $\kappa_\eta = \kappa_\eta(\phi)$ .

In  $k$ -distribution methods, gas absorption spectra are treated as random variables with given distribution functions. It can be noticed that this statistical view was almost inexistent in the early works on  $k$ -distribution methods, including Ambartsumian’s work [6]. It only became common when the notion of “correlation” between spectra was proposed to treat local variations in spectrum arising from gas non-uniformities [7]. “Correlation” is a concept which is fundamentally based on a probabilistic view of the problem.

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