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Improved stability criterion and output feedback control for discrete time-delay systems☆

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ABSTRACT

This paper investigates the stability and stabilization for a class of linear systems with time-varying delay. We provide a new finite-sum inequality which is a powerful tool for stability analysis of time-delay systems. Applying the inequality, a new stability criterion is proposed in terms of linear matrix inequalities (LMIs). We also design a method for static output feedback (SOF) control problems which contains two parts. The first part is to find an initial values of the matrix variables. By utilizing the initial values, the condition for SOF control problems can be solved by an improved path-following method. Numerical examples demonstrate the effectiveness of the stability criterion and the SOF stabilization method.

1. Introduction

Time delay is very common in industry processes, internet environment, economic and biological systems. The problem of delay-dependent stability analysis and stabilization for delay systems has received considerable attention in the past few decades.

In the field of stability analysis, the objective is to derive a effective delay-dependent stability criterion for determing an admissible maximum upper bound of delay. There have been many methods based on various tools to evaluate the stability for time-delay systems [1–4,28–30]. A substantial part of results is based on Lyapunov–Krasovskii functional method. One of the difficulty of this method lies in the bounds of the integrals that appear in the derivative for continuous-time systems, and of the finite-sum that appear in the forward difference for discrete-time systems. For continuous-time cases, most of the studies have been presented based on Jensen type inequality [5] because fewer decision variables are required than other existing methods. In [6], a new inequality based on Wirtinger inequality has been proposed which can obtain a tighter bound than Jensen inequality. Very recently, a free-matrix-based integral inequality [7,8] and an auxiliary function-based integral inequality [9] are proposed which include the Wirtinger-based inequality as their special case and yield less conservative stability criterion. For discrete-time systems, the Jensen-type method [15] is the most widely used method [10]. A free-weighting matrix method [13] can be regarded as an extension of Jensen-type method. An Abel-lemma-based [11] and

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the Seuret et al. [27] finite-sum inequalities are discrete-time versions of the Wirthinger-based integral inequality. Recently, the finite-sum inequalities in [23–26] can achieve less conservative results than the above methods.

On the other hand, the delay-dependent stabilization and controller design conditions cannot be expressed strictly in terms of LMIs. The CCL algorithm [21] is modified in [13] to handle the problem of designing SOF controllers for discrete time-delay systems. An augmented form of the closed-loop plant [19] is applied in [22] to establish an equivalent form of the stability criterion via an SOF controllers, and a iterative algorithm is proposed. The literature [12] presented a new SOF stabilization method with the gain matrix as a direct design variable. However, the above methods are still conservative. There is room for further investigation. The path-following method studied in [18] implies linearization method as its key step. It is improved by Chen et al. [20,32] by introducing a new linearization method, which enhanced the convergence and the search performance of the method. It is showing a huge advantage in SOF controller design problems.

This paper develops a new finite-sum inequality. Applying the inequality, we develop a new stability criterion for discrete-time systems with time-varying delay. Then, the criterion is extended to solve the SOF control problems. Two algorithms are built to solve the nonlinear matrix inequalities in the conditions. Firstly, we build an initial value optimization algorithm (IVOA) to find the initial values. By utilizing the initial values, the conditions can be solved by the improved path-following method.

The remainder of this paper is organized as follows. Section 2 is the problem formulation and preliminaries. Section 3 gives the new finite-sum inequality and a new stability criterion. Section 4 gives the method of SOF control design for discrete time-delay systems. Section 5 demonstrates examples and comparisons of the proposed methods with existing methods.

Notation: The symbol (*) represents a term that is induced by symmetry. I and 0 are the identity matrix and zero matrix with appropriate dimensions, respectively. The superscripts T denotes the matrix transpose. For any $A \in \mathbb{R}^{n\times n}$, we define $Sym\{A\} = A + A^T$. Let $e_i = [0_{n\times(i-1)n}\ I_n\ 0_{n\times(m-i)}]$, $i = 1, \dots, m$. Let the vectors $\omega \in \mathbb{R}^{3n}$, and $\bar{\omega} \in \mathbb{R}^{4n}$ with the definition $\omega^T = [\omega_1^T, \omega_2^T, \omega_3^T] = \left[x^T(-r_1), x^T(-r_2), \frac{1}{r+1}\sum_{j=-r_2}^{-r_1} x^T(j)\right]$, and $\bar{\omega}^T = [\omega_1^T, \omega_2^T, \omega_3^T, \omega_4^T] = \left[x^T(-r_1), x^T(-r_2), \frac{1}{r+1}\sum_{j=-r_2}^{-r_1} x^T(j), \frac{6}{(r+1)(r+2)}\sum_{j=-r_2}^{-r_1}\sum_{i=j}^{-r_1} x^T(i)\right]$, respectively. Let $r = r_2 - r_1$, $\eta(j) = x(j+1) - x(j)$, and $\bar{\eta}(j) = x(j) - x(j-1)$.

2. Problem formulation and preliminaries

Consider a linear discrete-time system with time-varying delay:

$$\begin{cases} x(k+1) = Ax(k) + A_d x(k-d(k)) + Bu(k), \\ y(k) = Cx(k) + C_d x(k-d(k)), \\ x(\theta) = \phi(\theta),\ \theta = -d_2, -d_2+1, \dots, 0, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $y(k) \in \mathbb{R}^p$ is the measured output, $\phi(\theta)$ is an initial condition, and the time delay $d(k)$ satisfies $d_1 \le d(k) \le d_2$, where d_1 and d_2 are positive integers. The matrices A, A_d, B, C, C_d are constant matrices of appropriate dimensions.

The following SOF controller is exploited

$$u(k) = Fy(k), \tag{2}$$

where $F \in \mathbb{R}^{m\times p}$ is a controller gain matrix for system (1). Combing (1) and (2), the closed-loop systems is obtained as

$$\begin{cases} x(k+1) = \hat{A}x(k) + \hat{A}_d x(k-d(k)), \\ x(\theta) = \phi(\theta),\ \theta = -d_2, -d_2+1, \dots, 0, \end{cases} \tag{3}$$

where

$$\hat{A} = A + BFC,$$
$$\hat{A}_d = A_d + BFC_d.$$

The following Lemma can be regarded as the discrete-time version of the inequality in [7].

Lemma 1 ([23,31]). *For positive definite matrix $R \in \mathbb{R}^{n\times n}$, symmetric matrices $Z_i \in \mathbb{R}^{3n\times 3n}$, any matrices $Z_{12} \in \mathbb{R}^{3n\times 3n}$, and $N_i \in \mathbb{R}^{3n\times n}$, $i = 1, 2$, satisfying*

$$\Phi = \begin{pmatrix} Z_1 & Z_{12} & N_1 \\ & Z_2 & N_2 \\ & * & R \end{pmatrix} \ge 0, \tag{4}$$

and integers r_1 and r_2 with $r = r_2 - r_1 \ge 1$, the following inequality holds:

$$\sum_{j=r_1}^{r_2-1} \eta^T(j) R \eta(j) \ge \omega^T \Omega_1 \omega, \tag{5}$$