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Routes to chaos from axisymmetric vertical vortices in a rotating cylinder

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In this work, we study several routes of the transition to chaos from a steady axisymmetric vertical vortex in a rotating cylinder depending on thermal gradients and rotation rates. The analysis is done using nonlinear simulations. For a fixed rotation rate, the chaotic regime appears, as thermal gradients increase, after a sequence of supercritical Hopf bifurcations to periodic, quasiperiodic and chaotic flows in a scenario similar to the Ruelle–Takens–Newhouse route to chaos. For moderate values of the rotation rate we find vortices that tilt and move away from the center of the cylinder in a periodic, quasiperiodic and finally chaotic movement around the central axis. For larger rotation rates the axisymmetric vortex splits into two symmetric vortices that move periodically around the central axis, and lose the symmetry merging again in one non-axisymmetric vortex that moves around the central axis quasiperiodically and later chaotically. The transitions to chaos when the rotation rate is varied at fixed thermal gradients reveal also the appearance of periodic, quasiperiodic and chaotic states in different routes. Tilted single vortices, double vortices and more complex structures with multiple vortices are reported in this case. The transitions are studied through a force balance analysis. Results are of interest as they connect to the behavior of some atmospheric vertical vortices.

1. Introduction

The transition of flows to chaos is a subject of great interest in fluid dynamics. Different routes to chaos are well-known: the Ruelle–Takens–Newhouse route [1,2] for which the flow becomes chaotic after three incommensurate bifurcations; the Feigenbaum scenario, for which chaos is reached after an infinite sequence of period-doubling bifurcations [3]; or the Manneville and Pomeau route for which the chaotic flow appears after an intermittency regime [4]. There is a dependence on the geometry, initial conditions, and other characteristics of the flow in the transition to chaos.

In Rayleigh–Benard convection, a sequence of instabilities takes place as the Rayleigh number increases leading the flow from laminar to chaotic. For small rectangular cavities the route to chaos in Rayleigh–Benard convection has been widely studied experimentally and numerically [5–9]. In Ref. [5] quasiperiodic flows with two or three frequencies and periodic-doubling bifurcations preceding the chaotic flow are found experimentally. In Ref. [10] quasiperiodic flows with four or five independent frequencies were observed. Experiments in horizontal cylindrical annuli show a Pomeau–Manneville route to chaos [11], and a route similar to the Ruelle–Takens scenario [12]. Numerically, an entire Ruelle–Takens route to chaos

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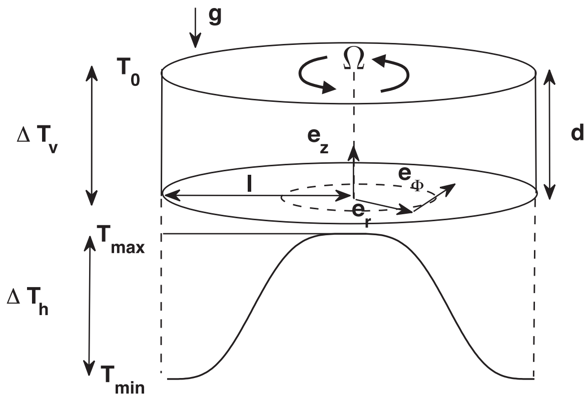

Fig. 1. Physical setup.

with a constant heat flux is followed in Ref. [13]. In vertical cylindrical annuli non homogeneously heated from below, authors study numerically the transition from a steady axisymmetric vertical vortex to a chaotic flow in a non-rotating frame of reference [14], revealing a route to chaos similar to that described by the Ruelle–Takens–Newhouse theory. In the evolution as the Rayleigh number increases, authors report the formation of subvortices embedded in the primary vortex, subvortical structures that strengthen and weaken, almost disappear before reforming again, by the effect of the thermal gradient increase.

In a rotating frame of reference, there are not many studies on the combined effects of thermal gradients (vertical and horizontal) and rotation. In Ref. [15] authors prove the generation of single-cell vortices and two-cell vortices (eyed-vortices) in the axisymmetric regime, and tilted single vortices and double vortices when the basic axisymmetric state bifurcates, depending on the thermal gradients and rotation rates. In the present work we extend results in [15] by studying the transition to chaotic flows as the Rayleigh number *Ra* and the Ekman number *Ek* vary. The numerical experiments performed reveal routes to chaos similar to that described in Ref. [14]. We find vortices that tilt, develop an eye and vary the intensity in their movement around the central axis, double vortices that develop and then merge in one tilted vortex, and more complex structures of several vortices. Different kinds of flows are reported in this study: geostrophic flows, for which the Coriolis force and pressure gradient force are almost in balance, and the inertial and viscous forces have no significant effect; gradient flows for which the centrifugal force together with the Coriolis force counter the gradient pressure; and cyclostrophic flows for which the inertial forces are found almost in balance with the pressure gradient forces, with the viscous force and the Coriolis force having a negligible effect.

Results presented are of interest for vortex dynamics, which is present in many atmospheric phenomena, such as dust devils, tornadoes, hurricanes or cyclones [16–19]. Observations reveal changes in the morphology of atmospheric vortices as the transition of a single-cell vortex to a double-cell vortex (vortex with an eye developed), or tilting of the vortex towards the direction of motion, very common in dust devils [16,20,21]. In larger-scale systems such as tornadoes, the formation of a multiple-vortex structure is reported [22,23], and of great interest is the formation of a double-eyed vortex found at Venus's south pole [24].

The paper is organized as follows. Section 2 presents the physical setup and the mathematical formulation of the problem in a dimensionless form. In Section 3 the numerical procedure for the temporal and spatial discretizations is introduced. Section 4 presents numerical results on flow behavior as the Rayleigh number *Ra* and the Ekman number *Ek* are varied. Finally, in Section 5, conclusions are presented.

2. Formulation of the problem

Fig. 1 shows the physical setup consisting of a horizontal fluid layer in a cylindrical container of radius l (r coordinate) and height d (z coordinate) in a rotating frame with a constant rotation rate Ω. At $z = 0$ the imposed temperature has a Gaussian profile which takes the value $T_{\max}$ at $r = 0$ and the value $T_{\min}$ at the outer part ($r = l$). The temperature at the upper surface is $T = T_0$. We define $\triangle T_v = T_{\max} - T_0$, $\triangle T_h = T_{\max} - T_{\min}$ and $\delta = \triangle T_h / \triangle T_v$.

In the governing equations, $\mathbf{u} = (u_r, u_\phi, u_z)$ is the velocity field, T is the temperature, p is the pressure, r is the radial coordinate, and t is the time. They are expressed in dimensionless form after rescaling: $\mathbf{r}' = \mathbf{r}/d$, $t' = \kappa t/d^2$, $\mathbf{u}' = d\mathbf{u}/\kappa$, $p' = d^2 p/(\rho_0 \kappa \nu)$, $T' = (T - T_0)/\triangle T_v$. Here $\mathbf{r}$ is the position vector, κ the thermal diffusivity, ν the kinematic viscosity of the liquid, and ρ_0 the mean density at temperature T_0. The domain in (r, θ, z) coordinates is $\mathcal{D} = [0, \Gamma] \times [0, 1] \times [0, 2\pi]$ where $\Gamma = l/d$ is the aspect ratio.

The non-dimensional equations for Boussinesq convection with rotation (with primes now omitted) are,

$$\nabla \cdot \mathbf{u} = 0, \qquad (1)$$