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First-passage failure of linear oscillator with non-classical inelastic impact

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ABSTRACT

The first-passage failure of linear oscillator with inelastic impact subjected to the additive and multiplicative random noises is investigated. The impact is described by the non-classical inelastic impact model, which is essentially different from the traditional impact model and can provide the whole information of the impact process. First of all, the impact force in the motion equation is replaced by the quasi-linear damping and nonlinear stiffness terms. Then, the stochastic averaging is adopted and the averaged Itô stochastic deferential equation of the total system energy is derived. Last, by solving the established backward Kolmogorov equation and Pontryagin equation from the averaged Itô equation numerically, the conditional reliability, the conditional probability density function (PDF) and the mean time of first-passage failure can be obtained. The comparison between the analytical results and those from Monte-Carlo simulation reveals the proposed procedure is effective. The influences of some system parameters are discussed in detail.

1. Introduction

The oscillator with impact, which is the so-called vibro-impact system, is an important system and widely exists in the structure and mechanical engineering, such as the offshore platform suffering the impact from the sea waves [1], the rotating machinery sometimes impacting into the barriers [2], the collision of ocean vessels [3]. The vibro-impact system can degrade system performance, and plays an important role in system safety. The main distinguish between the vibro-impact system and the ordinary system is the impact force, which is strong nonlinear and non-smooth. Besides the non-smooth behaviors, the existence of the random factors, such as random excitation [4,5], random material characteristic [6,7], noises in the measurement [8] will dramatically increase the complexity of the vibro-impact system. The significant engineering application and the complicated dynamical mechanism of the vibro-impact system have motivated the researchers to research them [9–12].

There are mainly two kinds of the impact model developed to describe the impact force, the classical impact model and the Hertzian contact model [13]. The classical impact model is the most popular one and can describe both the elastic and inelastic impact force. The mathematical description is $\dot{x}_+ = -r\dot{x}_-$ with $0 \leq r \leq 1$. The motion of the oscillator with impact contains the discontinuous boundary condition about the impact velocity $\dot{x}_+$ (velocity of the mass after collision) and rebound velocity$\dot{x}_-$(velocity of the mass before collision). Specially, $r = 1$ corresponds to the case with perfect elastic collision, while $r = 0$ corresponds to the completely inelastic collision. The non-smooth behaviors existed in the boundary condition make the system unsolvable and have to develop several transformations of state variables to eliminate the discontinuity. The most successful techniques have been proposed by Zhuravlev [14] and Dimentberg and Menyailov [15].

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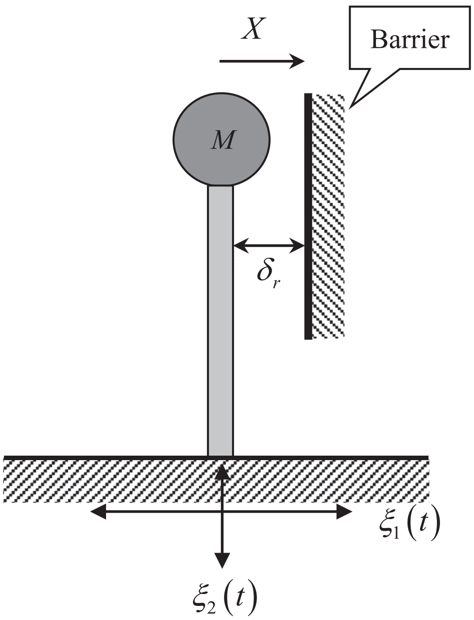

Fig. 1. Schematic of the linear oscillator with a right-side barrier.

With help of these powerful transformation techniques, the oscillator with impact transformed to the nonlinear system without any velocity discontinuity, which pinpoint the road to adopt the traditional asymptotic approaches. The other model is the so-called Hertzian contact model, in which the contact force is proportional to the 3/2 power of the relative displacement. Though the Hertzian contact model depicts the whole impact process, it is can only model the elastic impact process [16]. By using the stochastic averaging, the stationary solutions of SDOF and MDOF vibro-impact system subjected to the Gaussian white noise excitation have been investigated [17,18]. By perturbation approach, the stationary response of vibro-impact system under Poisson white noised has also been investigated [19].

Recently, non-classical inelastic impact model, a new impact model, has been adopted to capture the feature of the impact force. The non-classical inelastic impact model is developed from impact response experiments of elastic-plastic based on two assumptions: (i) the dynamic deformation mode is broadly similar to the quasi-static deformation mode, and (ii) the material is strain-rate insensitive [20]. This type of inelastic model can be used to describe the inelastic impact, and depict the whole impact process keeping the merits of the Hertzian contact model. Based on the non-classical inelastic impact model, the stochastic responses of the vibrating system with inelastic impact have been researched by the stochastic averaging technique [21]. In the field of the random dynamical system, the reliability associated with the system safety is another crucial problem [22]. For instant, the pipes collisions in nuclear industry will introduce the large inelastic deformation, which can always lead to the pipe leak and affect work safety [23,24]. Reliability analysis is to determine of the probability of the system in the safety domain, and the first-passage failure is one approach to analyze the system reliability. Unfortunately, except very special one-dimension diffusion processes, obtaining the solutions of the first-passage failure analytically are almost impossible. Therefore, many approximate methods have been considered to analyze the first-passage failure. Among all the approximate techniques, stochastic averaging can reduce the system dimension and is a powerful way to solve this problem [25].

In this manuscript, the reliability of the linear oscillator with non-classical inelastic impact subjected to additive and multiplicative random excitations is investigated. The inelastic impact is described by the non-classical inelastic impact model developed from the impact experiments. First, the energy dissipation balance is adopted to separate the complex impact force into the equivalent restoring force and energy-dependent damping, and then the equivalent reduced system without non-smooth impact force is obtained. According the theorem of Khasminskii [26], the total system energy, a slowly varying process, can be approximated to the Markov process and the associated averaged Itô equation can be derived. The establishing of the associated backward Kolmogorov equation and Pontryagin equation paves the way to the system reliability. Some numerical results are given and the influences of the system parameters are discussed in detail. The comparison with the results from the Monte Carlo simulation proves the accuracy of the proposed technique.

2. Linear oscillator with non-classical inelastic impact

In this paper, a column-mass system with a stop on the right-side is considered, as shown in Fig. 1. By using the Galerkin technique and only the first model is reserved, the general vibro-impact system with external and parametric