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Formulations and decomposition methods for the incomplete hub location network design problem with and without hop-constraints

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A B S T R A C T

The incomplete hub location problem with and without hop-constraints is modeled using a *Leontief substitution system* approach. The *Leontief* formalism provides a set of important theoretical properties and delivers formulations with tight linear bounds that can explicitly incorporate hop constraints for each origin-destination pair of demands. Furthermore, the proposed formulations are amenable to a Benders decomposition technique which can solve large scale test instances. The performance of the devised algorithm is primarily due to a new general scheme for separating Benders feasibility cuts. The novel cuts render a stabilizing effect that is directly responsible for the solution of instances up to 80 nodes.

1. Introduction

Hub-and-spoke networks have been a rich source of research for over a quarter century [8], since the seminal work of O'Kelly [45]. Hubs are facilities responsible for connecting and routing flow demands between many pairs of origin and destination (OD) nodes in a logistic or telecommunication network. They can function as switching/sorting/distribution and/or as consolidation/break-bulk centers. When operating as consolidation facilities, high volume carriers can be deployed to transport flows in bulks on inter-hub links, allowing thus scale economies to be exploited, one of the greatest appeals of hub-and-spoke systems.

Early hub location models [45–47] assume strict connection policies. OD demands traverse through at least one or at most two hubs. Hubs are assumed fully interconnected, whereas non-hub nodes are single or multiple assigned to hubs. Usually, no direct connections between non-hub nodes are allowed. Further, bulk transfers on inter-hub links have lower transportation costs due to the effects of scale economies represented by a constant discount scalar $0 \le \alpha \le 1$.

Though the aforementioned assumptions simplify the modeling and computational analysis, they create unrealistic networks for many cases, besides granting unjustifiable scale economies to quite small optimal flows [7]. To circumvent these

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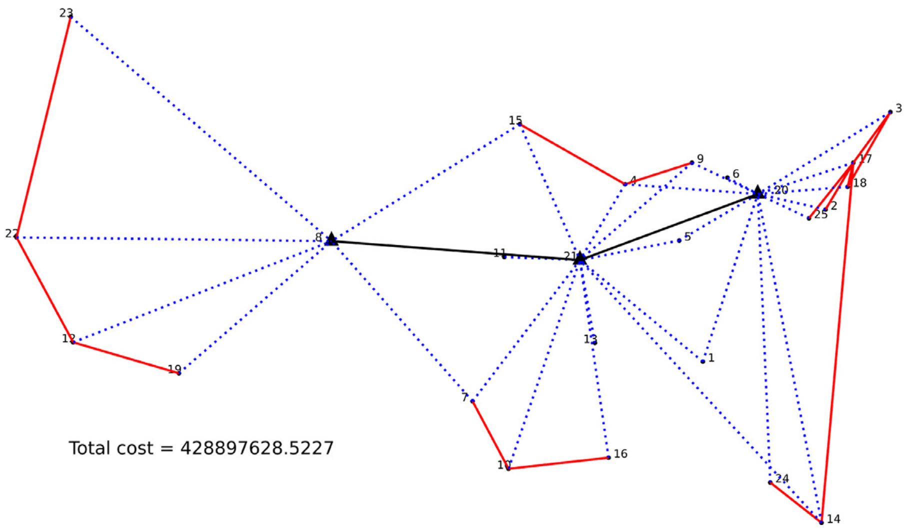

Fig. 1. p-hub constrained problem with $p = 3$. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

limitations, more interesting and flexible considerations with wider applicability have been assumed by the literature in the last decade (e.g. [1,9,11,12,17]) leading to *incomplete hub networks*.

In these networks, installed hubs are interconnected by only some of the available inter-hub links which are selected according to technological aspects of the addressed application or to the assumed cost components. While the former requires specific topological constraints during modeling, the latter requires far more elaborate formulations capable of yielding different underlying hub networks.

The research works developed by the authors in [1,11,12,16,18,29,31,55] introduce such technological constraints to satisfy applications' requirements. Whereas, in contrast, [48] recently describe a model with fixed and variable cost components for connections and hubs that is capable of generating many different network topologies depending on the relative magnitudes of the cost coefficients.

Though [48]'s formulation can be seen as a natural extension of the 3-index model of [23], it includes direct connections between non-hub nodes, and a better modeling for the scale economies. Nevertheless the large number of integer variables and the absence of topological restrictions typically used in hub location models to reduce the range of admissible solutions do not allow for problems with more than 25 nodes to be solved in a reasonable time on a regular computer.

Regardless of the modeling perspective, both technological or cost driven formulations of the literature oversimplify service level controls for OD demands. It is customary when designing incomplete hub networks to prescribe a number p of hubs to be installed to implicitly limit the maximum number $(p + 1)$ of connections or hops an OD flow has on its path. Restricting the number of sorting/connection points on an OD's path may improve the service level of a network.

Though easy to model, installing p hubs to implicitly limit the number of sorting points may yield into poor system designs. When p hubs are installed, at most $p(p - 1)/2$ inter-hub connections can be set up which significantly reduces the options for an OD flow to exploit scale economies. Most likely higher cost networks will be attained.

One way to properly handle service levels on hub location problems is to explicitly constrain the number of hops in a path, not the number of hubs to install for a network. Though the difference is subtle between both approaches, the hop-constrained one requires a much more ingenious modeling and resolution framework that leads to the design of more interesting, flexible, and lower cost hub systems.

To exemplify the aforementioned, Figs. 1 and 2 show the resultant incomplete hub networks for the same problem instance when the number of hubs to be installed is set to $p = 3$, and when OD demands' paths are actually hop constrained to $p + 1$ or 4, respectively. Red and black lines represent direct links between non-hub nodes, and inter-hub connections, respectively, whereas dashed blue lines assign non-hub nodes to hubs. Triangles depict hubs, while dots non-hub nodes.

The implicit approach (Fig. 1) has a higher total cost than the explicit hop-constrained one (Fig. 2). Note that only two inter-hub connections are installed in the former, in contrast with nine links in the latter. By having more inter-hub connections, the OD flows in the hop-constrained variant can better exploit scale economies, attaining thus lower costs. Notice also that Fig. 2 respects the limit of 4 hops for any OD path.

One might dispute that by setting $p = 6$, i.e. to install the same number of hubs of Fig. 2, one would obtain the same network. However Fig. 3 rebuts this claim by showing a totally different solution. Note that, on this particular solution, the total cost has actually decreased. On the other hand, several paths display more than 4 hops. In brief, hop-constrained paths yield a higher service level for the OD demands than installing a fixed number of hubs, because they induce different hub network structures with more inter-hub connections, allowing better exploitation of scale economies.