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Energy cycle of brushless DC motor chaotic system

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ABSTRACT

The vector field of the brushless DC motor (BLDCM) chaotic system is regarded as the force field of a pure mechanical system via the transformation of Kolmogorov system. The BLDCM force field is decomposed into four types of torque: inertial, internal, dissipative, and generalized external torque. The forcing effect of each term in the force field is identified via the analogue of the electrical and mechanical system. The BLDCM energy transformation of four forms of energy—kinetic, potential, dissipative, and generalized external is investigated. The physical interpretation of force decomposition and energy exchange is given. The rate of change of the Casimir energy is equivalent to the power exchanged between the dissipative energy and the energy supplied to the motor, and it governs the different dynamic modes. A simple and optimal supremum bound for the chaotic attractor is proposed using the Casimir function and optimization.

1. Introduction

Chaos applies not only to physical problems such as the trajectories of objects in turbulent media, but also to problems in biology, mathematics, chemistry, engineering, medicine, astronomy [1], music, business, geoscience and environmental science.

The existence of chaos in motor systems was first discovered by Kuroe in 1989 [2]. The mathematical model for a permanent magnet synchronous motor was first derived fitting for analysis of chaos and bifurcation [3]. The brushless DC Motor (BLDCM) is a type of synchronous permanent-magnet motor. In recent years, the BLDCM has achieved a brilliant expansion in the automotive, aerospace and household-appliance industries, robotics, food and chemical industries, electric vehicles, medical instruments, and computer peripherals [4,5]. The BLDCM is noted for its high efficiency, long life, low noise, and good speed-torque characteristics [4].

The BLDCM chaotic system was found in 1994 by Hemati [6]. In regard to performance, the occurrence of chaos in motors is highly undesirable in most engineering applications [7]. Thus, studies of chaos control have been performed on several types of motor drive systems [8,9]. The synchronization of the BLDCM chaotic system was also studied [10].

Regarding the analysis of chaotic models, the main research focus is on their dynamical analysis. Although some of the literature has reported on the application of chaotic systems in meteorology [11] and celestial chaos analog [1], most research has focused on the dynamics of the Lorenz system. The research themes usually are numerical calculation, aperiodic solutions, sensitivity to initial conditions, bifurcation theory [3,12], power spectra, circuit implementation, fractional order [13], chaos-based communication [14], chaos control and synchronization [15], generation of chaos [16], and existence of chaos [17]. However, these aspects of research aforementioned cannot reveal the mechanism or reason for the production of dynamic modes. To explore this aspect, the mechanics of chaotic systems must be investigated. The lines of study include

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force analysis and the energy transformation between internal energy and supplied energy. If chaos is investigated within mechanics, more of the fundamentals can be uncovered.

Arnold [18] presented a Kolmogorov system describing a dissipative-forced dynamic system or hydro-dynamic instability with a Hamiltonian function. For instance, in geoscience and environmental sciences, especially fluid dynamics, the Navier–Stokes equations (the Galerkin approximation) can be simplified as a Kolmogorov system in fluid dynamics [19]. Pasini and Pelino gave a unified view of the Kolmogorov and Lorenz systems [20,21], thereby providing the forcing analysis of the Lorenz system. Qi and Liang studied the mechanics of a four-wing chaotic system [22] and a 3-D chaotic system [23] through the transformation of Kolmogorov system to perform a forcing analysis. Furthermore, Qi and Zhang gave the analysis of energy cycling and bound for the 3-D chaotic system [24]. In this regard, the Hamiltonian function and the Kolmogorov system provide a starting point in studying the mechanism underlying these chaotic systems. The Casimir function, like enstrophy or potential vorticity in the context of fluid dynamics, is very useful in analyzing stability conditions and the global description of a dynamical system [25,26]. Both the four-wing chaotic system and the 3-D chaotic system were built from numerical simulation instead of physical derivation [27,28]. Even if the Lorenz system describing the atmospheric convection, but the Lorenz model was derived through a process [29]. However, the BLDCM model describes a real physical process of electromagnetic motor [6,9]. Therefore, the mechanical research result of the BLDCM is applicable and practical to the design and control of the system.

The number of parameters of BLDCM system is reduced for the Hemati model [6]. However, the modified parameters and variables lose their physical dimensions. Hence research outcomes using this model cannot match accordingly the dynamics of the original BLDCM system. This paper investigates the original physical model instead of the dimensionless one.

The presence of chaos in the BLDCM physically causes the system to oscillate and creates acoustic noise and mechanical vibration, thereby consuming electrical energy and reducing service life. On the other hand, chaotic mixing has been proposed to improve the energy efficiency and degree of homogeneity by using mechanical means that are essentially based on the design of permanent-magnet DC motor which serves as the agitator to produce chaotic motion [30–32]. Therefore, the studying of chaos for the system helps to avoid the generation of chaos, and also to create chaotic regulation so as to improve the configuration at the design stage.

In this paper, the original BLDCM is transformed into a Kolmogorov-type system, which decomposes the vector field of the function into inertial, internal, dissipative, and external torque. The kinetic energy and potential energy of the system are identified. We show that the rate of change of the Casimir energy is the exchange power between the dissipative energy and the supplied energy, and the exchange power determines the dynamic modes of the BLDCM and the supremum bound of the chaotic attractor. Different dynamical modes are revealed through combinations of energy.

Normally, it is difficult to find the bound of a chaotic attractor. Two important methods of finding the bound have been developed: one is Lyapunov-based method finding a positive definite matrix solve in the Lyapunov stable equation [33], another one is localization of compact invariant sets which is efficient to find ellipsoid bound [34,35]. However, we propose a new and simpler method based on rate of change of the Casimir function.

The paper is organized as follows: The mechanics of the BLDCM chaotic system is analyzed in Section 2. The energy cycling of the system and the boundary of the chaotic attractor are proposed in Section 3. Section 4 reveals the energy cycling mechanism underlying the different dynamic modes. A conclusion is given in Section 5.

2. Mechanics of BLDCM chaotic system

The equations describing the non-salient-pole (or called round pole or smooth air gap) BLDCM can be written via a Park transformation as [6,9]

$$\begin{aligned} L\dot{i}_q &= -Ri_q - nL\omega i_d - nk_t\omega + u_q, \\ L\dot{i}_d &= -Ri_d + nL\omega i_q + u_d, \\ J\dot{\omega} &= nk_t i_q - b\omega - T_L, \end{aligned} \tag{1}$$

where dot '.' denotes the derivative with respect to time t; i_q is the quadrature-axis current, i_d the direct-axis current, and ω the rotor velocity; R is the winding resistance, L_a self-inductance of the winding with $L = \frac{3}{2}L_a$, n the number of permanent-magnet pole pairs, k_e the coefficient of the motor torque with $k_t = \sqrt{3/2}\,k_e$, J the inertia of moment, b the damping coefficient of bearing; u_q and u_d are the voltage across quadrature-axis and direct-axis, and T_L the external torque due to load.

To reduce the number of parameters and make the system like the Lorenz system, Hemati [6] developed a model through a non-dimensionalization and obtaining the form [6]

$$\begin{aligned} \dot{\tilde{x}}_1 &= -\tilde{x}_2\tilde{x}_3 + \rho\tilde{x}_3 - \tilde{x}_1 + \tilde{u}_q, \\ \dot{\tilde{x}}_2 &= \tilde{x}_1\tilde{x}_3 - \tilde{x}_2 + \tilde{u}_d, \\ \dot{\tilde{x}}_3 &= \sigma(\tilde{x}_1 - \tilde{x}_3) - \tilde{T}_L. \end{aligned} \tag{2}$$

Because of this non-dimensionalization, the parameters and variables lose their original physical dimension. The variables in (2) do not physically correspond to those in the original system (1). ρ is a free parameter losing physical meaning in (2). $\sigma = \frac{\tau b}{J}$ is a combination of three parameters, where $\tau = \frac{L}{R}$ denotes the reciprocal of damping ratio, $\frac{b}{J}$ is the damping