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A model-order reduction method based on wavelets and POD to solve nonlinear transient and steady-state continuation problems

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ABSTRACT

We introduce a wavelet-based model-order reduction method (MOR) that provides an alternative subspace to Proper Orthogonal Decomposition (POD). We thus compare the wavelet and POD-based approaches for reducing high-dimensional nonlinear transient and steady-state continuation problems. We employ a global regularized Gauss–Newton (GN) algorithm for solving zero-residual problems on a reduced subspace. We rediscover that this latter is just a generalization of the Petrov–Galerkin method (PG) which retains GN's fast convergence rate. Numerical results included herein indicate that wavelet-based method is competitive with POD, for small rank systems (≈ 100) and compression ratios below 25% while POD achieves up to 90%. Full-order-model (FOM) results demonstrate that the proposed PGGN algorithm outperforms the standard PG method.

1. Introduction

Underbody blast simulations are highly complex and involve solving nonlinear algebraic systems of millions of equations and unknowns. Such simulations allow assessing the explosion impact on vehicles and personnel safety, as well as studying critical design configurations and enabling decision-making stages [1]. These parametric studies are intense from both CPU time and memory requirements. The CPU burden strongly suggests employing MOR techniques to perform simulations in real time that allow decision-making in a timely fashion. Different types of Reduced-Order Models (ROM) have been proposed to alleviate this computational burden [2].

Traditionally, MOR techniques are based on POD. They usually consist of the following: a computationally expensive “offline” stage is first executed, during which the FOM is adequately studied at carefully selected points in the input parameter space to compute a representative reduced subspace whose basis is used to obtain a ROM of the original large problem. Then, during the inexpensive “online” stage, the ROM is solved and its solutions expanded back onto the original space [3].

If the waiting time to obtain a solution is not a constraint, then a widely popular time-consuming strategy is to compute a family of solutions of the problem, for a suitable sample of input conditions, where every single solution is a “snapshot”. We then ensemble all snapshots as column vectors, to compute a compressed subspace via POD, which spans the FOM solution [4–6]. An important issue is that the projection basis, and hence the ROM, only contains information that is present in

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the ensemble of snapshots. Thus, a careful snapshot selection is critical to constructing a successful POD basis [7]. The standard POD implementation implies approximating the problem of interest in a fixed reduced subspace of global basis vectors. However, this is not convenient to tackle problems characterized by different physical regimes, parameter variations, or moving features such as discontinuities and fronts. Having a large number of snapshots to capture all these regimes/local features makes global POD impractical [8]. This drawback suggests seeking for adaptive approaches based on a local basis and alternatives to POD. Kerfriden et al. described in [9] a bridge between POD-based model order reduction techniques and the classical Newton/Krylov solvers. Their method overcomes some of the POD's drawbacks found on structural problems involving plasticity or damage: find an initial snapshot that is adequately enough to represent the solution of the damaged structure accurately as well as significant topological changes that may occur in the structure. They proposed a corrective tool for the adaptive MOR for such mechanical problems whose novelty lies in the fact that integrates corrections inside the POD projection framework. Ojo et al. [10] considered the Discrete Empirical Interpolation Method (DEIM) MOR approach for reducing a discretized nonlinear energy equation that arises from photovoltaic systems where each module contains several silicon cells. Their results showed that DEIM reduced the system size significantly while retaining the accuracy of the solution. A CFD-based aerodynamic design computational methodology that employs MOR as a surrogate evaluator is presented in [11]. The method builds local POD basis based on a zonal approach for resolving shock waves and improving the surrogate prediction in transonic flow. A current application of POD based MOR to large scale parametrized wave propagation problems is presented in [12]. Chinesta et al. reviewed in [13] the foundations and applications of the proper generalized decomposition (PGD), an alternative MOR technique that uses successive enrichments to approximate an unknown field variable. PGD avoids the complexity of standard grid-based discretization approaches, and thus circumvents the curse of dimensionality. PGD views the input space globally as coordinates of a higher dimensional space wherein an approximation can be computed at once.

We presented a new strategy for projection and MOR in previous works [14,15], which computes a reduced subspace using discrete wavelets that were able to reproduce the FOM's behavior. This approach does not require using snapshots and thus is suitable to be applied to both transient and steady-state problems with no parameter variation. The roots of compressed sensing are traced back to signal processing, image processing, digital signal, among others applications, in all of which discrete wavelets play a key role. It became natural to exploit the usage of wavelets in MOR [16–18]. A wavelet–Galerkin MOR method is proposed in [19] to study the behavior of tall buildings that are prone to wind-induced stochastic vibration. The analysis in the Daubechies wavelet domain could transform the original nonlinear coupled differential dynamic problem into a much simpler system of random algebraic equations. Goyal and Mehra [20] developed a fast diffusion wavelet method for solving parabolic PDEs. They employed the same operator for the construction of diffusion wavelet as well as for approximation of the PDE. Furthermore, these diffusion wavelets were used as MOR subspace. Alsmadi et al. [21] presented a new MOR technique based on an artificial neural network (ANN) prediction. They applied the ANN-based MOR for different scale systems with substructure preservation. ANN-based MOR is compared with classical MOR techniques and the simulation results confirmed the validity of the new method. A Wavelet-based MOR is applied to reducing distributed parameter systems in [22]. The proposed MOR technique uses multi-resolution methods to represent the system's multiscale and local behavior. Applications such as heat transfer along a flat plate and a packed-bed reactor demonstrated the approach.

The goal of the wavelet-based MOR is to provide an alternative subspace, out of the box, for those problems where there is no time to build a POD basis or when a global POD basis cannot adequately represent the local behavior. However, this alternative approach does not attain significant compression ratios that are common with POD as we find out here. To this end, we designed a Petrov–Galerkin direction to obtain solutions to zero-residual problems using wavelets [14], which is a quasi-direction within the GN method. We thus compare the snapshot- and wavelet-based approaches for transient and steady-state continuation problems. We also propose a regularized procedure to avoid singularities of the Jacobian and a globalization strategy to ensure convergence regardless of the initial point. Our test cases show that the proposed enhancements outperform the standard Newton method. We implement this algorithm as a MATLAB library for solving high-dimensional problems. Preliminary numerical results included here for a set of large-scale problems show the applicability of this library to problems of practical interest and its ability to reproduce the FOM's relevant features.

The remainder of the paper is organized as follows: Section 2 presents the mathematical models of the governing partial differential equations. Section 3 revises the proposed MOR method and also shortly discusses basic wavelet and POD properties. In Section 4, we present concrete numerical examples of the application of the PGGN algorithm. The last sections state concluding remarks, future work, and acknowledgments, respectively.

2. Mathematical models

We introduce two mathematical models in this section. We start with the 1-D Advection–Diffusion nonlinear parabolic equation; we then derive Bratu's 1-D problem from its particular steady-state case. We generalize Bratu's 1-D problem to higher dimensional spaces. We utilize central finite differences for the numerical discretization of Eqs. (1)–(3). Appendices A and B cover the discretization process in detail.