Applied Mathematical Modelling 53 (2018) 267–275

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Inference for the generalized exponential stress-strength model

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ARTICLE INFO

Article history:
Received 15 January 2017
Revised 9 August 2017
Accepted 6 September 2017
Available online 14 September 2017

Keywords:
Generalized exponential distribution
Stress-strength model
Generalized pivotal quantity
Coverage probability

ABSTRACT

This study develops inferential procedures for the generalized exponential stress-strength model. A generalized confidence interval for the stress-strength reliability is derived when the stress and strength variables follow the generalized exponential distributions with the common rate parameters. Moreover, based on the Fisher Z transformation, a modified generalized confidence interval for the stress-strength reliability with the unequal rate parameters is proposed. The performance of the proposed procedures is evaluated by Monte Carlo simulation. The simulation results show that the coverage percentages of the proposed generalized confidence intervals are quite close to the nominal coverage probabilities, even for small sample sizes. Finally, an example is used to illustrate the proposed procedures.

1. Introduction

The generalized exponential distribution was proposed by Gupta and Kundu [1]. The probability density function and cumulative distribution function (cdf) of the two-parameter generalized exponential distribution are given by

$$f(x,\alpha,\lambda)=\alpha\lambda\left(1-e^{-\lambda x}\right)^{\alpha-1}e^{-\lambda x},\quad x>0, \tag{1}$$

and

$$F(x,\alpha,\lambda)=\left(1-e^{-\lambda x}\right)^{\alpha},\quad x>0, \tag{2}$$

respectively, where $\alpha>0$, $\lambda>0$ are the shape and rate parameters, respectively. The generalized exponential distribution (1) will be denoted by $GE(\alpha,\ \lambda)$. For $0<\alpha\le 1$, the density function is a decreasing function and for $\alpha>1$, it becomes an uni-modal function. The generalized exponential distribution has increasing and decreasing hazard rate functions depending on the shape parameter α. It has been observed that the generalized exponential distribution can be used quite effectively to analyze skewed data set and it is a good alternative to the Weibull distribution. In many situations, the generalized exponential distribution might provide a better data fit than the Weibull distribution.

Statistical inference for the generalized exponential distribution has received the attention of some authors. Gupta and Kundu [2] compared various methods of estimating the parameters of the generalized exponential distribution using simulation method. Raqab [3] derived the exact expressions for means, variances and covariances of record statistics from the generalized exponential distribution. Jaheen [4] derived Bayes and empirical Bayes estimators for the unknown shape parameter

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http://dx.doi.org/10.1016/j.apm.2017.09.012
0307-904X/

of the generalized exponential distribution based on record statistics. Raqab and Madi [5] discussed the Bayesian inference for the generalized exponential distribution. Mitra and Kundu [6] discussed the parameter estimation for the generalized exponential distribution based on left censored sample. Chen and Lio [7] derived the point estimators for the generalized exponential distribution under progressive type-I interval censoring. Gu and Yue [8] discussed the existence and uniqueness of the maximum likelihood estimators (MLE) for the generalized exponential distribution. Yu et al. [9] derived the point and interval estimations for the generalized exponential distribution with left censoring. Gupta and Kundu [10] gave some reviews for the existing results and some recent developments for the generalized exponential distribution.

In the classical stress-strength model, both the stress X_1 and strength X_2 of the unit are assumed to be random, and the stress-strength reliability is defined as $R = P(X_1 < X_2)$. This model was first introduced by Birnbaum [11] and developed by Birnbaum and McCarty [12]. Since then, it has been widely used in many fields ([13–15]). Weerahandi and Johnson [16] considered hypotheses testing and interval estimation of the reliability R when the stress and strength variables are independent normally distributed. Kundu and Gupta [17] and Krishnamoorthy and Lin [18] considered the estimation of the reliability R for the Weibull distributions. Lio and Tsai [19] discussed the estimation of the reliability R for the Burr XII distributions based on the progressively first failure-censored samples. Recently, Baklizi [20] derived Bayesian estimation for the reliability R based on the exponential records. Condino et al. [21] gave the likelihood and Bayesian inferential methods for the reliability R using lower record values from a proportional reversed hazard family. Kotz et al. [22] provided some excellent information on past and current developments in the area.

When the stress and strength variables follow the generalized exponential distributions with the common rate parameters, Kundu and Gupta [23] derived the MLE and interval estimation for the reliability R. Baklizi [24] discussed likelihood and Bayesian inferences for the reliability R with the known common rate parameters based on lower record values. Wong and Wu [25] further studied the confidence interval for the reliability R based on lower record values from the generalized exponential distributions with the known common rate parameters by using the third-order approximation method. Our simulation results show that the coverage percentages of the interval estimation proposed by Kundu and Gupta [23] are not close to the nominal coverage probabilities. In addition, the assumption of the common rate parameters may be very restrictive in practice. Thus, it is necessary to consider the inferential methods for the reliability R without the assumption of the common rate parameters. The main aim of this paper is to focus on the inference of the reliability R with or without the assumption of the common rate parameters.

The remainder of the paper is organized as follows. In Section 2, we propose two estimators and a generalized confidence interval (GCI) for the stress-strength reliability R when the stress and strength variables follow the generalized exponential distributions with the common rate parameters. The finite sample properties of the proposed estimators and GCI are assessed by Monte Carlo simulation. In Section 3, we derive the MLE and two other estimators for the reliability R under the unequal rate parameters. On the basis of the Fisher Z transformation, we develop a GCI for the reliability R. The performance of the proposed procedures is also assessed by Monte Carlo simulation. In Section 4, an example is provided to illustrate the proposed procedures. Finally, we provide some final conclusions in Section 5.

2. The generalized inference for the stress-strength model with the common rate parameters

Suppose that $X_1 \sim GE(\alpha_1, \lambda)$, $X_2 \sim GE(\alpha_2, \lambda)$, and they are independently distributed. Then the reliability of the stress-strength model is defined as

$$\begin{aligned} R &= P(X_1 < X_2) = E[P(X_1 < X_2 | X_2)] \\ &= \int_0^\infty (1 - e^{-\lambda y})^{\alpha_1} d(1 - e^{-\lambda y})^{\alpha_2} \\ &= \frac{\alpha_2}{\alpha_1 + \alpha_2}. \end{aligned}$$

To derive interval estimation of the reliability R, the following lemmas are needed (see [9]).

Lemma 1. *Let $Z_1, \ldots, Z_n$ be a random sample from the exponential distribution with mean θ and $Z_{(1)} < Z_{(2)} < \cdots < Z_{(n)}$ be the corresponding order statistics. Furthermore, let*

$$\begin{aligned} S_i &= \sum_{j=1}^{i} Z_{(j)} + (n-i) Z_{(i)}, \ i = 1, \ldots, n, \\ T &= 2 \sum_{i=1}^{n-1} \log(S_n / S_i). \end{aligned}$$

Then (1) T and S_n are independent; (2) $T \sim \chi^2(2n-2)$ and $2S_n/\theta \sim \chi^2(2n)$.

Lemma 2. *Let*

$$f(\lambda) = \frac{\log(1 - e^{-b\lambda})}{\log(1 - e^{-a\lambda})},$$

where $b > a > 0$ are constants. Then $f(\lambda)$ is strictly decreasing on $(0, +\infty)$.