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Quasi-static interaction between non-uniform beams and anisotropic permeable saturated multilayered soils with elastic superstrata

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ABSTRACT

The soil above the water table is considered as elastic superstrata, and the soil below the water is regarded as a multilayered saturated soil with anisotropic permeability. The analytical layer element method is applied to obtain the fundamental solution of the soils, and the finite element method is used to solve the Timoshenko or non-uniform beam. Taking a Laplace transformation for the global matrix equation of the beam, the coupling equations for the soil–beam interaction are derived in the Laplace transformed domain by considering the displacement harmony conditions. By solving the coupling equations, the solution in the transformed domain can be achieved. With the aid of Laplace inverse transformation, the solution in the physical domain is obtained. Finally, the influence of the consolidation time and the thickness of the elastic superstrata as well as the application of varying cross-sections on the quasi-static interaction between the beams and the soils are investigated.

1. Introduction

In the field of industrial and civil construction, municipal engineering and water project, the foundation beam is a familiar foundation form. The elementary beam theory taking one general displacement into account was established by Bernoulli–Euler at the beginning of the 17th century, which neglects the effects of shear deformation on deflection. However, it is used widely owing to its simplicity for common beams. But with the increase of a beam's height-to-span ratio, the calculation accuracy of the elementary beam theory can not satisfy the needs of projects. Then the Timoshenko beam (TB) theory considering two general displacements was presented [1]. Selvadurai [2] took the opinion that the shear effect of a foundation beam has a significant influence on the settlement of soils. Essenburg [3] found that shear deformation could significantly influence the analysis of an infinite beam subjected to a concentrated force or a concentrated moment on a Winkler foundation. The main foundation models in the analysis of the interaction between beams and soils systems include the Winkler foundation, the two-parameter foundation and the elastic half-space foundation at present. For a TB on a Winkler foundation, Aydogan [4] proposed a stiffness matrix method for a beam element taking shear effects into account on a Winkler foundation, in which a second-order term of the derivative of the beam's deflection as the shear effect is considered. Yin [5] presented an analytical solution for an infinite Timoshenko beam subject to a point load on a Winkler foundation, and

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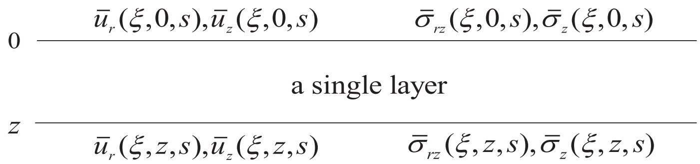

Fig. 1. The stresses and displacements for a single elastic soil layer.

established a finite element model for the same Timoshenko beam to illustrate that the shear deformation can't be neglected. As is seen from the researches above, studies on a Winkler foundation simplify the subgrade to some independent springs without considering their interaction. For a TB resting on a two-parameter foundation, Giger and Shirima [6] established the complete relationship between forces and displacements for a Timoshenko beam element on the basis of the exact solution of two differential equations for the problem. Onu [7,8] deduced the stiffness matrices of the TB on a two-parameter foundation taking the shear effect into account with the differential-equation approach. Although the interaction between springs is considered in a two-parameter foundation by a vertical spring constant of the Winkler foundation in combination with a horizontal linkage of the vertical springs, there may be difficulties in obtaining the parameters in practical engineering. For an elastic half-space, Zemochkin and Sinitsyn [9] presented the rigid bar method, using some hinged rigid bars to connect the beam and the foundation. Tullini et al. [10] utilized the coupled finite element method and boundary element method to investigate a TB element on an elastic half-space. However, limited by the fact that the foundation beams should be treated as three-dimensional in practical projects and the soils are naturally layered [11–16], foundations of a half-space assumption would be inaccurate. Considering the stratification of soils, Ai et al. [17] dealt with the analysis of the elastic foundation finite beams on 3D multilayered isotropic soils with a boundary element method. Later, Ai et al. [18,19] presented a theory combining finite element and analytical layer element to analyze a TB on elastic and poroelastic multilayered soils. In addition, in the case of long-span and multi-span framework structures, foundations with varying cross-sections are frequently used to adjust the uneven settlement and subgrade reactions. However, there are few studies about beams with varying cross-sections, so it is necessary to investigate the interaction program between beams with varying cross-sections and soils.

Soils are usually layered in nature, so it is of great importance to take the stratifications of soils into account. Besides, when considering ground water, soils usually exhibit anisotropic permeability and there is a certain thickness of elastic water-free soils. But few of the above articles take the stratification and anisotropic permeability of soils and elastic superstrata into account. In this paper, the foundation model of anisotropic permeable saturated multilayered soils with elastic superstrata is adopted, which conforms with the practical engineering better. On the other hand, the non-uniform beam with varying cross-sections is also analyzed in this study. Firstly, the analytical layer-element method [20–22] is realized to deduce the stiffness matrix of anisotropic permeable saturated multilayered soils with elastic superstrata. Then based on the principle of minimum potential energy, the element stiffness matrix of the TB bending in plane can be established. On the basis of the finite element method, the global matrix of beams is obtained by proper discretization [23–25], and the total stiffness matrix of the beam with varying cross-sections is established by the superposition principle of matrixes. By combining the conditions of the displacement compatibility of the soil–TB system, the final stiffness matrix of beams and foundation is achieved. Three examples are demonstrated to verify the accuracy of the proposed theory, and several other examples are carried out to investigate the influence of the consolidation time and the thickness of the elastic superstrata as well as the application of varying cross-sections on the quasi-static interaction between the beams and the soils.

2. Analytical layered-element solution for anisotropic permeable saturated multilayered soils with elastic superstrata

2.1. Analytical layered-elements of an elastic soil layer

The problem of axisymmetric elastic multilayered soils has been solved by Ai and Cai [18] using the analytical layer-element method. Starting with the governing equations and constitutive equations of axisymmetric elastic body, and based on the analytical layer-element method and the Hankel transformation, the relationship between the displacements and stresses of a single soil layer in Hankel transformed domain can be expressed as:

$$\begin{bmatrix} -\bar{\mathbf{\Gamma}}_1(\xi,0) \\ \bar{\mathbf{\Gamma}}_1(\xi,z) \end{bmatrix} = \mathbf{K}_1 \begin{bmatrix} \bar{\mathbf{U}}_1(\xi,0) \\ \bar{\mathbf{U}}_1(\xi,z) \end{bmatrix} \tag{1}$$

where $\bar{\mathbf{\Gamma}}_1(\xi,z) = [\bar{\sigma}_{rz}, \bar{\sigma}_z]^{\mathrm{T}}$ is the stresses vector, $\bar{\mathbf{U}}_1(\xi,z) = [\bar{u}_r, \bar{u}_z]^{\mathrm{T}}$ is the displacements vector; $\bar{\mathbf{\Gamma}}_1(\xi,0)$ and $\bar{\mathbf{U}}_1(\xi,0)$ are the values of $\bar{\mathbf{\Gamma}}_1(\xi,z)$ and $\bar{\mathbf{U}}_1(\xi,z)$ when $z=0$, respectively; $\bar{\sigma}_{rz}$, $\bar{\sigma}_z$ are the radial shear stress and vertical normal stress in the Hankel transformed domain, respectively; $\bar{u}_r$ and $\bar{u}_z$ are the radial displacement and vertical displacement in the Hankel transformed domain, respectively; ξ is the Hankel transform parameter, $\boldsymbol{K}_1$ is an exact and symmetric matrix of order 4×4 which establishes the relationship between the displacements and stresses in the Hankel transformed domain of a single soil layer as shown in Fig. 1, and its specific elements can be found in Ref. [18].