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Response of moist paperboard during rapid compression and heating

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ABSTRACT

The response of moist paperboard exposed to extensive compression and heating in short periods of time is investigated. A generic framework describing this response, in a thermodynamically consistent manner, has been derived previously. The present paper provides explicit formats of the necessary constitutive relations specific to moist paperboard exposed to extensive compression and heating in short periods of time. The transient transports of mass, momentum and energy, as well as specific interaction terms are considered for orthotropic paperboard. The elasto-plastic response is taken into account in a large strain setting. The exchange of mass between the water bound to the fibers and the water vapor during the sealing is also considered. Simulations of an idealized sealing of two sheets of paperboard are performed and the predicted distributions of temperature, vapor pressure, out-of-plane stress and Forchheimer number are studied. The discussion related to the results from the simulations provides a deeper insight to how the different transport processes will affect the paperboard and how these are coupled. The closed system of equations, including the explicit formats of constitutive relations, provided in this paper makes it possible to set up suitable experiments for validation of the model.

1. Introduction

Paperboard may be considered as a material composed of a solid phase, defined by a dry fiber network, a liquid phase, defined by both liquid water bound to the fiber network and “free” liquid water that does not interact with the fiber network, and a gas phase which may be regarded as a mixture of two miscible constituents, namely water vapor and dry air. The presence of water in paperboard has a great influence on the response of the board. The amount of liquid water in the board is characterized by the moisture ratio, which is defined as the fraction between the mass of the liquid water and the mass of the dry fiber network. Neglecting the mass of the moist air inside the board the moisture ratio is equivalent to the dry basis moisture content. In the present paper, moisture ratios below the maximum hygroscopic moisture content (HMC) are considered and all liquid water is assumed to be bound to the fiber network, see Ref. [1]. Due to the manufacturing process, paperboard may be considered to be an orthotropic material, a property that introduces direction-dependent transports of mass, momentum, and energy. The characteristic directions in paperboard are: machine direction (MD), cross machine direction (CD), and out-of-plane direction (ZD), cf. Fig. 1.

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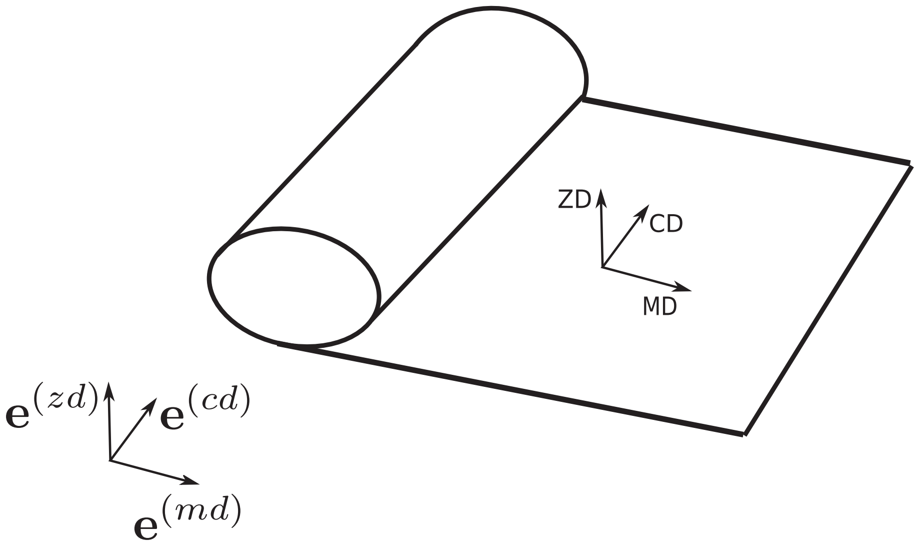

Fig. 1. The characteristic directions of paperboard.

The purely mechanical quasi-static stress–strain response of paperboard has been modeled in, e.g., [2,3], where the invariance properties discussed in [4] are adopted and where local characteristic directions are introduced in the form of director vectors which are assumed to evolve with the deformation of the board. The model presented in [2] describes the in-plane stress response of paperboard as elasto-plastic and the out-of-plane stress response as elastic, limiting the applications of the model to predominant in-plane loading. A more general model of the stress–strain response of paperboard is proposed in [5] and later extended in [6] to also include elasto-plastic out-of-plane response. To model the out-of-plane plastic response, the yield surface presented in [2] is enhanced with six additional sub-surfaces associated with the stress response in the out-of-plane direction. The explicit dependencies of moisture and temperature on the stress–strain response of the fiber network are not considered in the present paper and the model suggested in [6] is adopted. The reader is referred to the work in [7,8] for more information on the explicit dependencies of moisture and temperature on the mechanical response of paperboard.

High velocity gas flow through paper hand sheets has been studied in [9] by assuming a nonlinear laminar flow described by the Forchheimer equation. The viscous and microscopic inertial resistances of hand sheets are investigated and the microscopic inertial losses are found to contribute more than 50% of the pressure loss. Liquid water transport through paper-like material have been studied in [10,11], where the swelling of the fiber matrix is included via an additional sink term in the balance of mass equation and it is shown that the model is able to predict the wicking experimentally produced in [12]. Intra-fiber and inter-fiber moisture diffusion in bleached kraft board are investigated in [13] and a model is derived which is able to predict the transient moisture content during a change in the ambient humidity. Transient heat transport in paperboard has been studied in, e.g., [14], where the out-of-plane thermal conductivity and the specific heat of commercial copy paper sheets are investigated. It is shown that both a series–parallel model and a lumped parameter model may be used to predict the density dependence of the out-of-plane thermal conductivity of copy paper sheets.

The coupled mass and heat transport in paper has been studied in, e.g., [15], where a hybrid mixture theory (HMT) framework is adopted to derive a model predicting the response of paperboard rolls during a change in the ambient humidity. In [16], balance laws, derived in [17,18], for transport in porous media are adopted and the evolution of the moisture and temperature distributions in printing and copying paper are analysed during printing. A three phase model describing the moisture, temperature and pressure distributions during a drying phase in the production process of cardboard has been derived in [19], and validated against experimental data in [20].

In order to better understand the response of paperboard to folding and filling processes, a generic framework has been developed in [21] that describes the transient transport of heat and moisture in deforming paperboard. The balance laws presented in [21] are derived from a HMT framework [22,23], and considers the balance of mass, the balance of linear momentum, the balance of energy of all components of the paperboard.

In the current paper, specific constitutive relations valid for moist paperboard are derived. A brief presentation of the governing balance laws is given in Section 3. After this introduction, it is shown, in Section 4, how the stress–strain response model described in [6] may be incorporated in the framework. Explicit formats of the constitutive relations specific for paperboard in environments similar to those present during the folding and filling process of a food package are derived in Sections 5, 6 and 7. Having retrieved a complete set of constitutive relations, simulations of an idealized sealing are presented and discussed in Section 8.

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