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Hamiltonian Boussinesq formulation of wave–ship interactions

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ABSTRACT

In this paper a new approach is described for the fully nonlinear treatment of the dynamic wave–ship interaction for potential flows. A major reduction of computational complexity is obtained by describing the fluid motion in horizontal variables only, the surface elevation and the potential at the surface. In such Boussinesq type of equations, the internal fluid motion is not calculated, but modeled in a consistent approximative way. The equations for the wave–ship interaction are based on a Lagrangian variational principle, leading to the formulation of the coupled system as a Hamiltonian system. With the ship position and orientation as canonical coordinates, the canonically conjugate momentum variables are the sum of the ship momemta and the fluid momenta. A beneficial consequence of this is that the momentum exchange between fluid and ship will be described without the need to calculate the pressure, which simplifies the numerical implementation of the equations considerably. Provided that the potentials with mixed Dirichlet–Neumann data can be calculated, the presented ship dynamics can be inserted in existing free surface flow solvers.

1. Introduction

In this paper the Hamiltonian description of inviscid free surface waves will be extended in a straightforward consistent way to include ship–wave interaction. For inviscid, irrotational flows with a free surface, the Hamiltonian formulation of the continuity and Bernoulli's equation at the surface uses the potential ϕ and the surface elevation η as canonical conjugate variables; this result based on contributions by Luke [1], Zakharov [2], Broer [3] and Miles [4] is summarized in Section 2.2. The dependence of ϕ, η on horizontal variables alone makes that the Hamiltonian formulation is of Boussinesq-type, i.e. dimension reduced.

For the coupling of a rigid ship with waves, Lagrangian and Hamiltonian methods have been used before, and the general equations are being used in numerical implementations. The use of Lagrangian principles for solid–inviscid fluid interaction started with Thomson and Tait, and Kirchhoff, see Chapter 6 of Lamb [5]. Miloh [6] seems to be the first to include the radiation potential to describe linearized surface elevation caused by forced objects. In Van Daalen [7] a concise derivation of the Lagrangian and Hamiltonian formulation is given for the water–ship interaction in incoming waves. Nowadays, the

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Lagrangian–Hamiltonian formulation seems standard, and has been extended to the coupling of water motion and flexible bodies by Xing and Price [8]; see this paper also for references to the recent literature.

Yet, listing the total set of the dynamic equations combined with the various Laplace problems that have to be solved, may somewhat hide the variational structure behind the equations, which may, at best, cause that not full advantage is taken of the peculiarities of the special structure. This is particularly evident in the treatment of the momentum exchange between ship and fluid, and the role the pressure is given. In the Hamiltonian formulation of this paper, it is found that *the dynamic change of the canonical momentum, which is the sum of the rigid body and the fluid momentum, equals the static hydrodynamic force*. Written in a Newtonian way for one motion component, the canonical momentum is $p_s + p_f$ where p_s is the ship momentum; p_f is the fluid momentum, i.e. the integral over the wetted ship hull of the fluid potential. The Hamilton equation is then of the form, writing τ^H for the hydrostatic forces:

$$\frac{d}{dt}(p_s + p_f) = \tau^H. \tag{1}$$

Note that dp_f/dt is (minus) the dynamic part of the pressure; in this Hamilton equation it is forced by the hydro-static part of the pressure. In the explicit time stepping procedure that we will derive these two momentum parts are treated differently.

However, in contrast, characteristic of seemingly all treatments of wave–ship interaction is the fact that this balance law is described by a formulation that *the change in the ship momentum equals the fluid forces acting on the ship*; see for instance the formulation for the 'Dissel' software by Lin and Kuang [9–11]. In the simplified notation this reads:

$$\frac{d}{dt}(p_s) = \frac{d}{dt}(-p_f) + \tau^H, \tag{2}$$

where the right hand side is now the total pressure. With this formulation, an explicit time stepping gives a different result; an implicit scheme will lead to the known problems to update the pressure to the same time level as the potential itself. It will be shown that the Hamiltonian formulation with explicit time stepping avoids the calculation of the pressure, and will consistently respects the order in which forces work.

The description of the Hamiltonian formulation of the coupled system in Section 3 finds its basis in work of Van Daalen [7], see also Van Daalen et al. [12]. The main modifications presented here concern the explicit expression of the equations in the Hamiltonian variables instead of using the pressure, which leads to the time evolution that avoids the calculation of the pressure.

Concerning the interior flow that has to respect Dirichlet and Neumann conditions, the total potential will be split in a way as described e.g. by Cummins [13] and Ogilvie [14]. One component of the total potential is the fluid potential with normal derivative vanishing at the ship and Dirichlet value at the free surface; it describes the tangential flow along the ship. The other part is the instantaneous radiation potential that is the sum of normalized instantaneous radiation potentials multiplied by the corresponding velocities for each of the six degrees of freedom. The evolution of the impulsive radiation potential is part of the Hamiltonian evolution, without the necessity to consider the convolution as described in Cummins, see Section 4.3.

Solving the interior potentials to obtain their effect on the free surface and at the ship hull can be done by using any reliable Laplace solver. The capabilities of the Laplace solver will determine directly the applicability of the wave–ship system, such as simulations above bathymetry, in harbors, and for ships with small under keel clearance. Results of the Laplace solver can be used directly in the governing Hamilton equations. The wave potential has to be updated each time step; for simplified 'linearized' ship interaction, it is sufficient that the normalized radiation potentials (and related added-mass coefficients) are calculated only once with the ship in its hydrodynamic equilibrium position. When aiming to keep the advantage of efficient calculations based on the dimension reduction in the Boussinesq description, the calculation of the effect of the potentials at the free surface and at the ship will have to be based on an approximation of the interior flow, which requires an extension of the methods used in Boussinesq models for free surface flows. In forth coming publications we will show the performance of the coupled system using Boussinesq reduction in two models of HAWASSI software [15].

The organization of this paper is as follows. In the next section we set notation and briefly describe the derivation of the Hamiltonian formulation for the free surface flows. The generalization to include water-body interaction will be described in Section 3. We restrict to the case of one free floating ship; the case of more or moving ships is a rather direct extension. In Section 4 the potentials and momenta that have to be calculated are summarized, and the time stepping for the Hamiltonian evolution is described. Conclusions are formulated in the final section.

2. Hamiltonian wave dynamics

The dynamical system for the combined wave–ship motions will be an extension of the Hamiltonian description of free-surface waves which will be shortly described in this section. Just as in Classical Mechanics for interacting rigid bodies, the total energy is the main quantity, expressed in canonical variables that form the state variable that has to be evolved in time.

In the first section we set notation, followed by the Hamiltonian formulation for waves only. The Lagrangian description for the wave–ship interaction, from which the Hamiltonian formulation for the water-body interaction will be derived in Section 3, is a generalization of the free surface Lagrangian. In the last section we deal with the practical aspect how to obtain the required kinetic energy as explicit expression in the canonical variables for waves above flat and varying bottom.