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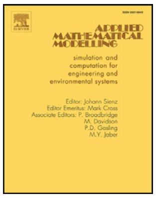

Flow models and numerical schemes for single/two-phase transient flow in one dimension

Xiaoju Du*, Ole Jørgen Nydal

Department of Energy and Process Engineering, Norwegian University of Science and Technology
Visiting address: Kolbjørn Hejes v 1B, Varmetekniske laboratorier
Postal address: NTNU, Institutt for energi-og prosessteknikk, Kolbjørn Hejes v 1B, NO-7491 Trondheim

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ABSTRACT

In the two-phase flow field, a traditional mathematical model for simulating the transition of severe slugging flow presents a challenge when liquid slugs completely block pipelines. Accordingly, an advanced and practical slug model that is derived from a mixture model associated with a slip closure is essential to solving the problem in cooperation with the two-fluid model. The model can offset numerical instability that arises from the discontinuous function of the friction factor across the transition from one flow pattern to the other. Two numerical schemes, the non-iterative and the iterative, are developed, and the proposed schemes can stably predict the transient problems under the Courant–Friedrichs–Lewy (CFL) condition for semi-implicit/implicit schemes. In the present work, pressure transients produced by a complex phenomenon, named water hammer effect, are captured using the single-phase flow model in one-dimension to verify the applicability of the numerical schemes and the friction factor model. At last, the analysis of the two-phase transient flow in a pipeline-riser system indicates that the significant advantage of the present schemes is the robustness that the numerical prediction of the severe slugging behaviour is accurate and stable.

1. Introduction

Various two-phase flow patterns exist widely in daily life and the oil/gas industry. Among them, the slug flows resulting in large pressure gradients and flow rates can be difficult to handle in many process industries [1,2]. The generation of slug flows contains different mechanisms: pigging slugs, hydrodynamic slugging, terrain slugging and riser-based slugging (also known as severe slugging). The severe slugging flow can occur periodically in pipeline riser systems, where long bubbles push the liquid slugs containing dispersed bubbles along the pipelines.

Slug flows, especially severe slugging flow, have been the subject of many experimental, theoretical, and numerical investigations. In the early years, many studies were dedicated to the modelling of severe slugging in pipeline riser systems, but with limited ability [2,3]. Also, a flow regime dependent two-fluid model, called the OLGA model, was developed by Bendiksen et al. [4] to simulate pressure transients in two-phase slug flow. Based on the transient two-fluid model, [5,6] obtained new correlations for the drag coefficient and the complex virtual mass force in the slug flow regime, and the non-physical

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* Corresponding author.
E-mail addresses: duxju@alumni.ntnu.no (X. Du), ole.j.nydal@ntnu.no (O. Jørgen Nydal).

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oscillations in the predictions are improved for simulating various steady-state and transient flows in pipes. Moreover, simulation and experimental results are compared in three test cases of steady and transient slug flow within horizontal pipes and slightly inclined (+5°) pipe [7]. [8] also performed numerical simulations on transition of slug flow using a large number of pipe configurations, which include horizontal, inclined, and V-section pipes. And a transient three-phase 1D simulator is developed for capturing transient slug flow [9]. The difficulty of offsetting numerical instabilities is complicated in a complex pipe configuration (including large inclinations) by the intermittent nature of slug flows. Consequently, more investigation is needed to attain a fully satisfied inclined flow model and robust numerical schemes.

In the present work, an appropriate friction factor model is applied for performing the upwind schemes to predict the pressure transient and damping in the water hammer effect [10]. A slug model, associated with the slip closure and the mixture two-phase flow model, has been derived to simulate slug regimes in an s-riser system. Numerical instabilities that arise from the discontinuous function of friction can be eliminated if the friction factor behaves continuous across the transition. Besides, the presented numerical schemes strengthen the robustness of the model as most of the existing numerical methods struggle to predict the formation of liquid slugs stably and accurately. More specifically, numerical methods are particularly vulnerable when the liquid completely blocks the pipe [11]. The fact that the gas disappearance in the lower section of pipeline leads to non-physical values and numerical oscillation can cause calculation failure. The proposed algorithm in this work exhibits a remarkable superiority in numerical stability.

The developed numerical schemes include the iterative scheme and the non-iterative scheme. In the iterative scheme, the numerical calculation repeats until the numerical errors are negligibly small at one time level; and the non-iterative scheme proceeds gradually one time step after another while the numerical errors are added as source terms in the linearised equations. Two test problems are studied in this work. The first-order upwind scheme is adopted to simulate the classical water hammer effect (single-phase flow) using a proper friction factor model; following this, numerical predictions of the severe slugging (liquid–gas) flow forming in an S-riser system are presented to demonstrate the ability of the proposed numerical schemes and the slug model.

2. Governing equations for transient flow

The one-dimensional governing equations for compressible isothermal single- and two-phase flow are presented in this section. Also, advanced concepts for friction factors are introduced and integrated into the flow models for solving specific flow problems.

2.1. Single-phase flow model

The single-phase model is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = S_m \tag{1}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2 + P}{\partial x} = G + f_w u, \tag{2}$$

where ρ is the density, u is the velocity, P is the pressure, and S_m is the mass source. In the momentum source terms, gravitational force G equals to $-\rho g$ where g is the gravitational acceleration; the friction force is expressed as $f_w u = -\lambda \rho |u| u / 2D$ (λ is the friction factor and D is the pipe diameter).

To characterise flow regimes—laminar flow, turbulent flow, and transitional flow—a dimensionless number, Reynolds number, is used for flow in the pipe:

$$Re = \frac{uD}{\nu}, \tag{3}$$

where ν is the kinematic viscosity of the fluid. Normally, a steady friction factor is applied in the traditional mathematical model and it varies according to different flow regimes. First, the steady friction factor λ_s is equal to $64/Re$ for laminar flow ($Re < 2300$); for turbulent flow ($Re > 4000$), it is expressed as $0.3164 Re^{-0.25}$; and for transitional flow regime, the friction factor is estimated as the maximum of the above two values.

However, the pressure waves generated in the water hammer case cannot be reproduced numerically with only the steady friction factor λ_s. Hence, it is necessary to introduce an unsteady friction factor, and the friction factor becomes:

$$\lambda = \lambda_s + \lambda_{us} \tag{4}$$

Brunone et al. [12] proposed an unsteady friction model as follows:

$$\lambda_{us} = \frac{K_B D}{u|u|} \left(\frac{\partial u}{\partial t} - C \frac{\partial u}{\partial x} \right), \tag{5}$$

where K_B is a dimensionless coefficient and C is the speed of sound for liquid. This unsteady friction model is well-established with the method of characteristics, but not with finite volume methods [10]. Another friction factor model [13] is

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