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Numerical study on viscoelastic fluid flow past a rigid body

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ABSTRACT

Viscoelastic non-Newtonian fluids can be achieved by adding a small amount of polymer additives to a Newtonian fluid. In this paper, numerical simulations are used to investigate the influence of such polymer additives on the behavior of flow past a circular cylinder. A numerical method is proposed that discretizes the non-linear viscoelastic system on a uniform Cartesian grid, with a penalization method to model the presence of the cylinder. The drag of the cylinder and the flow behavior under the effect of different Reynolds numbers (Re), Weissenberg numbers (Wi) and polymer viscosity ratios (ε) are studied. Numerical results show that different flow characteristics are exhibited in different parameter zones. The polymer viscosity ratio plays an important role at low Weissenberg and Reynolds numbers, but as the Reynolds and Weissenberg numbers increase, the influence of ε weakens. The drag force of the cylinder is mostly affected by the Reynolds and Weissenberg numbers. At low Reynolds numbers, the drag of the cylinder and the flow fields are only affected by a large value of Wi when the elastic forces are strong. Non-trivial drag reduction occurs only when there is vortex shedding in the wake flow, whereas drag enhancement happens when the vortex shedding is inhibited.

1. Introduction

When a tiny amount of soluble polymer with high molecular weight dissolves in a Newtonian solvent, such as water, the diluted solution exhibits complex mechanical characteristics, especially elastic behavior. Various physical phenomena can be observed that stem from the simultaneous presence in the fluid of elastic, viscous and inertial forces in different ratios. For example, elastic turbulence can be obtained and used for mixing and heat transfer in microfluidic devices when the inertial forces are negligible [1], while effective suppression of turbulence when the viscous forces are weak can reduce frictional drag by up to 80% [2,3]. Intense study of such flow structure modifications through the addition of polymer additives is motivated by both their wide industrial applications and their fundamental importance for understanding complex flow.

A polymeric molecule (polymer) is a long chain composed of a large number of repeated monomers. A typical drag-reduction polymer may include as many as 10^6 monomers. As a result of thermal agitation, a polymer put in a homogeneous flow can be treated as a statistically spherical coil. The radius of gyration of the polymer, R_0, which is the average size

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of the statistical sphere, is related to the number of monomers N by $R_0 = N^{0.6}a$, where a denotes the length of a single monomer according to ref. [4]. Typical values of R_0 are of the order 0.1 μm, which is generally smaller than the Kolmogorov dissipation scale in turbulence. By contrast, in an inhomogeneous flow, the polymer molecule is stretched into an elongated shape and can be characterized by its end-to-end distance R. Simultaneously, such an elongated molecule tends to relax to its equilibrium configuration as a result of the Brownian effect. After the shear is excluded, the characteristic time for an elongated molecule to recover to its equilibrium configuration is called the relaxation time and is an important index for viscoelastic solutions. According to Zimm [5], a convenient measure of the relaxation time for a linear chain is given by $\tau = \mu {R_0}^3/(k_B T)$, where μ is the solvent viscosity, k_B is the Boltzmann constant and T is the solution temperature. Furthermore, one of the most important dimensionless numbers in the mechanics of viscoelastic fluids is the product of the polymer relaxation time τ and the deformation rate of the flow $\dot{\gamma}$, known as the Weissenberg number; i.e. $Wi = \tau\dot{\gamma}$.

The elastic stress is defined as an elastic component proportional to the polymer deformation tensor of the viscoelastic solution. The elastic stress can be dominated by the elongation of the polymer molecules. Polymer elongation can in turn be limited by the polymers' reaction to the flow, as polymer can reduce the elongational rate of the flow. This results in a dynamical equilibrium state characterized by a constant average elongation that depends on the flow structure, the polymer concentration and polymers' molecular characteristics.

The molecular viscosity of the fluid is the result of chaotic movement and collisions among the fluid molecules. There is a significant increase in the molecular viscosity when a small amount of long-chained polymer is added to the solution. In contrast to a Newtonian fluid, the viscosity of a polymer solution may or may not vary with the shear rate, but most polymer solutions are inherently shear thinning (i.e. the viscosity decreases with the shear rate). This is because a segment of the molecular chain aligned with the flow produces no drag and therefore no contribution to the shear stress. The polymers' stretching and flow alignment increase with the shear rate, so a polymer solution is more-or-less shear thinning [6]. However, when a polymer solution is extremely dilute, the variation of the viscosity with the shear rate can be ignored. In this work, only a Boger fluid (an elastic fluid with constant viscosity) is modeled.

The variety of the available molecular structures means that polymers have a broad range of mechanical characteristics, resulting in inconsistent conclusions on their effects on drag in experiments. For example, a polymer with a linear molecular structure will be more effective for drag reduction than a branched polymer of the same molecular weight [7]. In addition, adding small amounts of polymer to a Newtonian fluid will make the solution viscoelastic. The total stress is proportional to both the deformation rate under the effect of viscosity (viscous stress) and the amount of deformation under the effect of elasticity (elastic stress). A viscoelastic solution can be thought of as a hybrid of both characteristics. Viscoelastic fluids therefore behave differently to Newtonian fluids. The viscoelasticity of polymer solutions can dramatically modify the flow behavior, such as changing the stability of laminar motion or the transition to turbulence, modifying the structure of vortices or vortex shedding and influencing the transportation of mass, momentum and scalars [8,9].

Drag reduction through polymer addition can be applied to reduce the skin friction drag of an internal flow, such as flow through a pipe or channel, in order to increase the flow rate for the same pumping cost, or it can be used to reduce the drag of a moving body, such as ship, to achieve a higher speed and reduce energy costs. While the former application has been widely studied, the latter has only recently attracted attention. Polymer additives can be circulated through fluids in internal flows (e.g. in pipes) to achieve a homogeneous concentration; however, it is difficult to sustain steady polymer levels in an external flow (e.g. water flowing past a ship at sea), which limits some relevant industrial applications. A promising new technology that mixes a fine powder of polyethyleneoxide (PEO) with self-polishing anti-fouling paint has been developed to reduce drag in external flow situations, such as the flow of water around ship hulls [10]. In this paper, the influence of the polymer viscosity ratio on the behavior of external flows is evaluated by numerical simulation.

External flows past obstacles, such as a circular cylinder, have been studied extensively for Newtonian fluids because of their extensive practical applications, e.g. the tubes in tubular and pin-type heat exchangers, which are used extensively for cooling electronic components, and in food and polymer processing applications. On the other hand, such cases also present rich opportunities for fundamental research: the flow over a circular cylinder exhibits diverse flow phenomena, such as boundary layer flow separation, wake flow, vortex shedding, vortex-induced oscillations, aerodynamic loading, momentum balance, lift and drag forces, shear layers, boundary layer transition and so on. The flow over a circular cylinder has therefore been studied as both a benchmark and a legitimate research case. Hence, studying the flow around a circular cylinder is fundamental to comprehensively understanding other complex flows over bluff bodies or rod bundles.

Despite the importance of such flows, the drag of a cylinder in viscoelastic flow has only recently attracted researchers and been studied numerically in a wide range of parameter spaces. In our previous study, a large number of numerical simulations in a wide parameter space were performed. The numerical results revealed that the drag of a cylinder could be enhanced or reduced in viscoelastic flow. The drag variation mostly depended on the value of the Reynolds number and Weissenberg number. The drag behavior could therefore be represented by a phase diagram in Wi-Re space for a polymer viscosity ratio of 0.01 [11-13]. As the polymer concentration increases, both the Weissenberg number and polymer viscosity ratio increase at the same time. The extra elastic stress induced by polymer extension is also expected to increase accordingly. However, it has not been well studied how the polymer concentration affects the flow, even though it seems intuitive that the elastic stress should increase as the concentration increases. The present paper aims to evaluate the effect of the polymer concentration on flow around a circular cylinder by varying both the Weissenberg number and the polymer viscosity ratio.