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Three-dimensional Green's functions of thermoporoelastic axisymmetric cones

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A B S T R A C T

On the base of Biot's consolidation theory, we introduce the steady-state general solutions of thermoporoelastic axisymmetric media initially. Several three-dimensional problems of porous media, such as the apex of a solid cone and a hollow cone under the action of a point fluid source and a point heat source in a steady state, are analyzed afterwards, respectively. By introducing the potential functions with the coefficients determined in line with the corresponding liquid-heat-force equilibrium relations and boundary conditions, we obtain the coupled fields of thermoporoelastic cones suffered from the point sources. Furthermore, the numerical examples as well as the contour plots of the coupled fields of thermoporoelastic cones are presented. The numerical results show that the phenomenon of stress concentration can occur near the point of action. The results associated with the apex angle $\pi/2$ are of importance to be used for constructing the analytical solutions of the boundary value problems as well as the defect problems.

1. Introduction

Due to their unique features of high specific surface area, acoustic and thermal insulation and excellent penetrability, porous media have a wide range of applications in industrial and manufacturing engineering. Geohydrology is one of the subjects involving porous media by utilizing the contemporary scientific methods earlier. Geologic materials such as soils are natural porous media; the studies of the coactions of the underground water and its saturated poroelastic aquifer are the indispensable parts of hydraulics [1], and are also common topics concerned in the fields of hydrogeology, underground hydrology and exploitation of the ground water resources.

The porous medium's theory has affected most of engineering fields at present, for instance, biological science, filtration technique and soil consolidation, and so forth, which will have a significant impact on modern industry and science development. In 1941, Biot [2] set up the three-dimensional (3D) consolidation model based on elastic theory and fundamental equations of porous media. As the continuous progress in the computational technology, Biot's consolidation theory has been applied to engineering practices far and wide, which sets the stage for the further studies on the fluid–solid coupling problems. Lo et al. [3] derived the analytical solutions for the dynamic response problems in a semi-infinite fluid-bearing porous medium. Hou et al. [4] derived the solutions of Green's functions of poroelastic bi-materials. For the heat storage and thermal pollution of the aquifer, the heat effect of porous media should be taken into account. The fluid–solid coupling

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problems considering temperature effect [5–7] have been brought to the forefront, involving in the excogitation for heat supply pipeline, underground storage of level radioactive nuclear waste and exploitation of geothermal reservoir etc. Biot [8] introduced the Boussinesq–Papkovitch functions that satisfied the Laplace equation and heat conduction equation, and obtained the general solutions of isotropic porous media. The general solutions of thermoporoelastic materials are derived by Li et al. [9], which played a basic role to the researches on Green's functions of the materials. Lu et al. [10] put forward the refined theory of a thermoporoelastic beam by the Lur'e method. By using the Laplace–Hankel transform, Ai and Wang [11] analyzed the thermal consolidation problems of multilayered thermoporoelastic media.

Green's function represents the generated fields by a point source with definite boundary conditions and initial conditions. Green's function is of great importance in theoretical and applied researches of hydraulics, being a base for further theoretical analyses and numerical calculations for hydrologic problems that the solid phase and fluid phase of poroelastic aquifer affect mutually. The two-dimensional (2D) or 3D Green's functions of infinite or semi-infinite planes [12–14], infinite or semi-infinite bodies [15–18], bi-materials [19–23], and multilayered structures [24,25] have been obtained, respectively. Cones are as one of the most basic structures, have been studied by many scholars. Panferov [26] analyzed the deformation problem of a transverse isotropic elastic cone subjected to axial forces and temperature loading. Ding et al. [27] analyzed the deformation of spherically isotropic cones subjected to concentrated loads. Ding et al. [28] obtained the solutions of a piezoelectric cone under concentrated loads. Popov and Vaysfel'd [29] solved the oscillations of an infinite cone in a steady state by means of the integral transformation method. Gurijala et al. [30] analyzed the vibration problems in a plane angular sector of a poroelastic elliptic cone.

The problem of cones subjected to concentrated loads is a typical problem in the elastic theory; moreover, that for considering multi-field coupling effects has caused an extensive concern. However, the thermoporoelastic problem for cones subjected to concentrated loads has not been researched. Considering the fluid-saturated transversely isotropic thermoporoelastic axisymmetric cones as the study objectives, this paper presents its 3D Green's functions in a steady state systematically. By introducing the potential functions and using the axisymmetric thermoporoelastic general solutions, the fundamental solutions of a solid cone and a hollow cone under the action of a point fluid source and a point heat source are obtained, respectively. The contour plots of their coupled fields are presented. Moreover, the influences on the coupled fields of the solid cone with the different apex angle and effective stress coefficient are analyzed.

2. General solutions for a thermoporoelastic medium

In the cylindrical coordinate system, the expressions of the strain tensor components of axisymmetric thermoporoelastic media are

$$\begin{aligned} \varepsilon_{rr} &= u_{r,r},\ \varepsilon_{\theta\theta} = u_r/r,\ \varepsilon_{zz} = u_{z,z},\ \varepsilon_{rz} = (u_{r,z} + u_{z,r})/2, \\ \varepsilon_{r\theta} &= \varepsilon_{z\theta} = 0, \end{aligned} \tag{1}$$

where the subscript prime "," denotes the derivative with respect to the space variables; u_ς ($\varsigma = r, z$) and ε_{jn} ($j, n = r, \theta, z$) are the displacement and strain components, respectively.

The constitutive equations of a fluid-saturated transversely isotropic thermoporoelastic medium in terms of the cylindrical coordinates are [31]

$$\begin{aligned} \sigma_{rr} &= C_{11}\varepsilon_{rr} + C_{12}\varepsilon_{\theta\theta} + C_{13}\varepsilon_{zz} - \alpha_1 P - \beta_1 T, \\ \sigma_{\theta\theta} &= C_{12}\varepsilon_{rr} + C_{11}\varepsilon_{\theta\theta} + C_{13}\varepsilon_{zz} - \alpha_1 P - \beta_1 T, \\ \sigma_{zz} &= C_{13}\varepsilon_{rr} + C_{13}\varepsilon_{\theta\theta} + C_{33}\varepsilon_{zz} - \alpha_3 P - \beta_3 T, \\ \sigma_{zr} &= 2C_{44}\varepsilon_{rz}, \end{aligned} \tag{2}$$

where σ_{jn} are the stress components; P and T denote the pore pressure increment and temperature increment, respectively; C_{il} ($i, l = 1, 2, 3, 4$) are the elastic constants with $C_{66} = (C_{11} - C_{12})/2$; α_1 and α_3 are the Biot's effective stress coefficients; β_1 and β_3 are the thermal modules.

Fluid flow and heat conduction in porous media follow the Darcy law [32] and Fourier law, respectively

$$\begin{Bmatrix} q_r \\ q_z \end{Bmatrix} = -\begin{bmatrix} \kappa_{11} & 0 \\ 0 & \kappa_{33} \end{bmatrix}\begin{Bmatrix} P_{,r} \\ P_{,z} \end{Bmatrix}, \quad \begin{Bmatrix} h_r \\ h_z \end{Bmatrix} = -\begin{bmatrix} \lambda_{11} & 0 \\ 0 & \lambda_{33} \end{bmatrix}\begin{Bmatrix} T_{,r} \\ T_{,z} \end{Bmatrix}, \tag{3}$$

where q_r and q_z are the fluid fluxes; h_r and h_z are the heat fluxes; κ_{11} and κ_{33} are the permeability coefficients; and λ_{11} and λ_{33} are the heat conduction coefficients.

The equilibrium equations are

$$\begin{aligned} &\sigma_{rr,r} + \sigma_{zr,z} + (\sigma_{rr} - \sigma_{\theta\theta})/r = 0, \\ &\sigma_{zr,r} + \sigma_{zz,z} + \sigma_{zr}/r = 0. \end{aligned} \tag{4}$$

Assuming that the thermoporoelastic loading changes slowly with time in porous media, the rates of entropy and fluid mass content both disappear. Consequently, in a steady-state, the fluid conservation and heat conductivity equation are respectively reduced to [7]

$$\kappa_{11}(P_{,rr} + P_{,r}/r) + \kappa_{33}P_{,zz} = 0, \quad \lambda_{11}(T_{,rr} + T_{,r}/r) + \lambda_{33}T_{,zz} = 0. \tag{5}$$